College and Career Readiness Standards-in-Action

Foundational Unit

2

WORKSHOP MATERIALS
MATHEMATICS

THINKING ACROSS LEVELS TO CONNECT LEARNING

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TABLE OF CONTENTS

Directions for Participants

1. Identify the progression topic to which each color group of standards belongs: fluency with operations, expressions and equations, or real-world applications.

2. Begin with the blue cards. Use knowledge of how concepts and skills build on one another to organize the color-coded cards in a logical order of progression from the lowest to highest level.

3. Use knowledge of the CCR Standards and the Unit 1 resource, Major Work of the Levels, to help identify the level (A, B, C, D, or E) for each standard on a fluency card.

4. Share results with others at your table, and discuss any points of agreement and disagreement.

5. Repeat steps 2 through 4 for the yellow and green cards.

Resource: Major Work of the Levels[1]

Color key: Black = Number, Red = Algebra, Blue = Geometry,
Green = Statistics and Probability

Level A (CCSS Grades K-1/Beginning ABE):
Number: Developing understanding of whole number place value for tens and ones
Number: Developing understanding of addition and subtraction and the properties of these operations
Geometry: Describing and reasoning about shapes and their attributes
Geometry: Developing understanding of linear measurement

Level B (CCSS Grades 2-3/ABE I):
Number: Extending understanding of base-10 notation
Number: Adding and subtracting to 1,000; fluency and application to 100
Number: Understanding multiplication and division of whole numbers to 100
Number: Understanding division as inverse of multiplication; single-digit divisors
Number: Developing understanding of fractions, especially unit fractions
Geometry: Using standard units of measure for length, time, liquid volume, and mass
Geometry: Developing understanding of area and its relationship to addition and multiplication
Geometry: Analyzing and partitioning 2-dimensional shapes

Level C (CCSS Grades 4-5 + 6/ABE II):
Number: Extending the number system to positive rational numbers
Number: Extending place value understanding for decimals to thousandths
Number: Attaining fluency with operations, using multi-digit whole numbers and decimals
Number: Understanding fraction equivalence and comparison
Number: Developing fluency with sums and differences of fractions
Number: Connecting ratio and rate to whole number multiplication and division
Algebra: Writing, evaluating, and interpreting expressions and equations
Geometry: Developing understanding of the coordinate plane
Geometry: Classifying geometric 2-dimensional figures based on properties
Geometry: Developing an understanding and solving problems involving volume and surface area
Statistics and Probability: Developing understanding of statistical variability

[1] This document is not meant to be a substitute for the CCR Standards for Adult Education; rather, it is meant to be used in conjunction with the CCR Standards for Adult Education, where full descriptions of the major work can be found in the introductions for each level.

Level D (CCSS Grades 6 + 7-8/ABE III):
Number: Extending number sense and fluency with operations to all rational numbers
Number: Understanding ratio and rate and using them to solve problems
Algebra: Applying proportional relationships
Algebra: Working with expressions and linear equations
Algebra: Solving linear equations and systems of linear equations
Algebra: Developing the concept of function
Algebra: Graphing functions in the coordinate plane and analyzing their graphs
Geometry: Solving problems involving scale drawings
Geometry: Solving problems involving 2- and 3-dimensional figures: area, surface area, and volume
Geometry: Analyzing 2- and 3-dimensional shapes using side length and angle measurements, similarity, and congruence
Geometry: Applying the Pythagorean theorem
Statistics and Probability: Understanding patterns of association for bivariate data and describing them with a linear equation, when appropriate
Statistics and Probability: Summarizing and interpreting data and data distributions
Statistics and Probability: Understanding and applying probability concepts
Statistics and Probability: Drawing inferences about populations based on random samples (probability distributions)

Level E (CCSS Grades 9-12/ASE I and II):
Number: Extending understanding of number systems to the set of real numbers
Number: Writing equivalent expressions involving radicals and rational exponents
Number: Reasoning quantitatively and the use of units and appropriate levels of precision
Algebra: Defining, evaluating, comparing, and modeling with linear, quadratic, and exponential functions and equations
Algebra: Building, interpreting, and analyzing functions using different representations
Algebra: Reasoning with and solving linear, quadratic, and exponential equations and linear inequalities
Algebra: Interpreting and using the structure of expressions to solve problems
Algebra: Operating with algebraic expressions, including polynomials and rational expressions
Geometry: Applying similarity and congruence concepts to geometric figures, including triangles
Geometry: Using geometric models and volume formulas to solve measurement problems
Statistics and Probability: Summarizing, representing, and interpreting one- and two-variable data, including using frequency tables

Resource: Color-coded Standards Cards (one page for each set)

- **Fluency With Operations**
- **Expressions and Equations**
- **Real-World Applications**

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Fluently multiply multi-digit whole numbers using the standard algorithm.
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all non-negative rational numbers.
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
Solve multi-step word problems posed with whole numbers and having whole -number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.	Analyze and solve pairs of simultaneous linear equations.
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Directions for Facilitators

1. Make a set of color-coded standards cards (one page for each set). These are on pages 5 - 7.

   - Fluency With Operations: on blue card stock (see page 5)
   - Expressions and Equations: on yellow card stock (see page 6)
   - Real-World Applications: on green card stock (see page 7)

2. Cut out the color-coded standards to have ready to distribute to participants.

Answer Key: Thinking Across Levels to Connect Learning

Theme 1: Fluency With Operations (blue cards)

Level	Standard	Content/Concept/Keywords
A	**1.OA.6**	Add/subtract within 20 …
B	**3.NBT.2**	Add/subtract within 1000 …
B	**3.OA.7**	Multiply/divide within 100 …
C	**4.NBT.4**	Add/subtract multi-digit wholes …
C	**5.NBT.5**	Multiply multi-digit wholes …
C	**6.NS.3**	Add/subtract/multiply/divide multi-digit decimals …
D	**7.EE.4a**	Solve word problems leading to equations … ($px + q = r$, etc.)
E	**A-APR.1**	Operate with polynomials

Theme 2: Expressions and Equations (yellow cards)

Level	Standard	Content/Concept/Keywords
A	**1.OA.2**	Solve word problems that call for … (add three numbers to 20)
B	**2.OA.1**	Use addition/subtraction within 100… (using an equation, etc.)
B	**3.OA.3**	Use multiplication/division within 100 … (using an equation, etc.)
C	**4.OA.3**	Solve multi-step word problems posed with … (represent using equations)
C	**6.EE.7**	Solve real-world and mathematical problems … (using linear equations)
D	**7.EE.4**	Use variables to represent quantities … (in a real-world problem)
D	**8.EE.8**	Analyze and solve pairs … (systems of linear equations)
E	**A-CED.1**	Create equations and inequalities … (to solve problems)

Theme 3: Real-World Applications (green cards)

Level	Standard	Content/Concept/Keywords
A	**1.OA.2**	Solve word problems … (involving sums to 20 of three whole numbers)
B	**2.OA.1**	Use addition/subtraction within 100 … (to solve problems)
C	**4.OA.3**	Solve multi-step word problems posed with whole numbers …
C	**6.NS.1**	Interpret and compute quotients of fractions … (and solve word problems)
D	**6.RP.3b**	Solve unit rate problems …
D	**8.EE.8c**	Solve real-world and mathematical problems … (leading to linear system)
E	**N-Q.1**	Use units as a way to understand … (to solve problems)
E	**A-CED.1**	Create equations and inequalities … (to solve problems)