VIDEO RESOURCE PACKAGE: OBSERVING MATHEMATICS CLASSROOMS

Videos Feature Instructors:
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Standards-in-Action 2.0

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TABLE OF CONTENTS

MATHEMATICS CLASSROOM OBSERVATION FOR RICHMOND ONOKPITE

What follows is an observation of a lesson taught by Richmond Onokpite. He served as an instructor from the Academy of Hope Adult Public Charter School in Washington, D.C. This observation provides concrete examples of what challenging state academic standards in mathematics look like in daily planning and practice. The observation tool is designed as a professional development tool for instructors, those who support instructors, and others working to implement standards. It is not designed for use in evaluation.

Core Action 1. Lesson content is rigorous and relevant for the level defined by the state-adopted standards.

Core Action 2. Learning activities are cognitively demanding and maximize opportunities for students to master the lesson content.

Core Action 3. Lesson content productively engages students.

Core Action 4. Lesson content is intentionally sequenced to develop students' skills and knowledge.

Core Action 5. Students' levels of understanding are checked throughout the lesson, and instruction is adjusted accordingly.

Core Action 1. Lesson content is rigorous and relevant for the level defined by the state-adopted standards.	**Y, N, or N/A**
A. Instructor presents a lesson with well-defined standards-based goals that focuses on the major work of the level (MWOTL).[1]	Y
B. Instructor presents a lesson that addresses the Standards for Mathematical Practice that are central to the lesson goals and connected to the targeted content.	Y
C. Instructor, when addressing the MWOTL, intentionally targets one or more aspects of rigor as appropriate for the addressed standard(s). Mark the aspect(s) of rigor the lesson addresses: • Conceptual understanding • Procedural skill and fluency • Application	Y

Evidence observed:

Indicator A:

Two CCR standards are listed in the lesson plan and reflected in the lesson:

- A central standard is 8.F.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- Another is 5.G.1: Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis. The second number indicates how far to travel in the direction of the second axis. With the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

Both standards identified in this lesson are central to students' learning.

Just as the standards require, Richmond addresses the concept of a function as a rule that assigns each input to exactly one output numerous times. This leads to evidence of student understanding. He encourages them to understand and use the coordinate plane for plotting ordered pairs.

[1] MWOTL – Major work of the level (MWOTL) – The most important mathematics in preparing adult learners for success in college, careers, and life. Instructors should learn to identify the topics that are—and are not—major topics for the various standards' levels. When planning instruction, they should focus deeply on the MWOTL rather than racing to cover topics in a mile-wide, inch-deep curriculum. The standards recently adopted by states require educators to significantly narrow and deepen the way time and energy is spent in the math classroom. When educators apply a MWOTL approach to teaching and learning, students can gain strong foundations in critically important areas: *conceptual understanding, procedural skill and fluency, and the ability to apply what they learn to solve math problems inside and outside the classroom.* Students should spend most of their time on the MWOTL; it should be the major focus of instruction. https://lincs.ed.gov/publications/pdf/ccr/Math_Unit_1 Materials/Math_1_part_mat.pdf

During the warm-up, Richmond asks students to identify a rule and complete a table with missing inputs for corresponding outputs, and vice versa.

Richmond shows students that a function graph is a set of ordered pairs consisting of an input and corresponding output. While Richmond doesn't always use language from the standard, his focus is on students grasping the concepts before introducing new vocabulary terms or definitions.

Richmond introduces students to the concept of graphing functions [at 00:22:53:00]. He communicates how to plot points and briefly introduces the terms "x-axis," "y-axis," and "coordinates," yet steers students away from using them. His strategy encourages students to use the "input" and "output" language rather than fall back on the often-misunderstood x and y terminology.

Richmond underscores the goals listed in his lesson plan throughout the class session. For example, he reviews how they have represented a function so far (using a rule or a table). Then says they will look at a third way [at 00:19:44:00]. When showing a graph later, he reinforces these three ways to show a function, that is with a table, a rule, and a graph [at 00:28:24:00]. He also clarifies the connection between the inputs and outputs (the ordered pairs) and the graph [at 00:26:00:00]. He points out the importance of the position of the origin of the graph [at 00:45:33:00].

Indicator B:

Named and reflected in the lesson are MP.8 (Look for and express regularity in repeated reasoning) and MP.3 (Construct viable arguments and critique others' reasoning). Here are two examples of Richmond eliciting these habits of mind in students:

- MP.8—During the warm-up, students are asked to complete several function tables. This helps them learn how to recognize the rule for a function based on the pattern observed in the inputs and outputs [at 00:00:25:00].
- MP.3—In the small- and large-group discussions during and after the warm-up and in the graphing activity, students are asked to present and support their findings. They also recognize errors, and discuss others' reasoning [at 00:05:34:00 and 00:37:49:00].
- Richmond specifically asks students whether an answer is correct and to justify their responses [at 00:11:17:00].
- He also asks them to critique and discuss each small group's graph [at 00:39:00:00].

Indicator C:

Two aspects of rigor are targeted in the lesson: conceptual understanding and procedural skill and fluency. Richmond asks questions throughout the lesson to check student conceptual understanding. Here are some examples:

- After the warm-up, when students report out, Richmond checks students' understanding [at 00:05:34:00].
- When introducing graphs of functions, Richmond first demonstrates with an example in front of the class. Then he has students apply the concepts to another example in small groups to become more secure and gain conceptual understanding [at 00:22:40:00 and 00:31:34:00].
- During the graphing activity where students are working in small groups, Richmond checks on students' understanding [at 00:32:49:00 and 00:34:33:00].
- When each group posts its graph at the front of the room, the class analyzes and discusses the graphs with Richmond's guidance [at 00:37:49:00].

The small-group work also provides students with opportunities to build their procedural skill and fluency in creating graphs from a rule or table.

Core Action 2. Learning activities are cognitively demanding and maximize opportunities for students to master the lesson content.	**Y, N, or N/A**
A. Instructor presents high-quality questions and tasks to prompt students to discuss their developing thoughts and elaborate on and justify their responses.	Y
B. Instructor consistently uses explanation, modeling, or examples to make the mathematics of the lesson explicit.	Y
C. Instructor provides students with opportunities to work with and practice level-specific problems and exercises.	Y

Evidence observed:

Indicator A:

The questions and problems presented by Richmond are high quality and challenging. Rarely could students answer questions posed with just a “yes” or “no.” Richmond frequently encourages students to elaborate on and justify their responses. Richmond often asks probing questions to help move along students’ thinking throughout the class period. Here are just a few examples:

- During the warm-up, when a group of students is stuck on a problem, Richmond asks questions that move them forward in their thinking. It also lets them complete the task [at 00:04:15:00].
- Richmond asks the group to explain how they got from the input to the output, and someone says “multiplying.” Even though this answer isn’t correct, he asks students to justify the answer rather than simply correcting it. Eventually, they come up with the correct answer [at 00:06:12:00].
- One student states that the answer is decreasing and gives an amount by which it is decreasing. Someone else at another table has a different answer. Richmond writes both answers on the board, and the class talks through which one is correct [at 00:08:08:00].
- During each group’s presentation of its graphs, he gives students the opportunity—and encourages them—to ask probing questions, requiring groups to explain and justify their graphs [at 00:37:52:00 and 00:39:19:00]. He doesn't immediately say whether the graph is right or wrong. Richmond asks several questions to pinpoint possible issues with the graphs, and he invites students to follow up on one another's comments.

Indicator B:

When Richmond introduces the concept of representing functions as equations, he first uses language that students are familiar with: “input,” “output,” and “rule” [at 00:11:55:00]. Then, he introduces using variables instead of these terms to start to form equations. After a few examples, he has students create examples. He also asks many questions throughout. Moreover, when students work in small groups or as a class, Richmond often asks questions rather than just delivering information. He often says, "I'm not telling you what I believe." Also, each time he asks the class to comment on a graph, he says, “I haven’t said whether this is correct or wrong.”

Indicator C:
The problems and activities presented are appropriate for Low/Middle Intermediate-level students. They match the standards at that level. Moreover, a variety of problems are offered. Included is a warm-up activity where students use a rule to complete a table. Also an activity where students graph a function, and another where students create a rule and a function table based on a graph. All students appear to be engaged in each activity. They discuss and ask questions throughout the lesson that is evidence that the problems and tasks are at the appropriate level.

Core Action 3. Lesson content productively engages students.	Y, N, or N/A
A. Students participate actively in sustained class discussions and activities where they build on each other's observations and insights.	Y
B. Students have varied opportunities to apply what they are learning in authentic adult-oriented contexts.	N
C. Most students display persistence with tasks and problems.	Y

Evidence observed:

Indicator A:

The class consists overwhelmingly of class discussions and group work. Even when Richmond presents new content, he makes sure students are actively involved. When introducing the concept of creating a statement or an equation based on a function rule, he consistently asks questions and engages students [at 00:11:50:00]. This is also true when students are reviewing the problems and activities. Each group has the opportunity to present its work creating a graph. Richmond talks for roughly one-third of the class time, while students talk for the other two-thirds.

All students participated in the activities throughout the lesson. Several students ask questions, and almost all students share their work and answers. Richmond calls on the quieter students at different times to bring more of them into the discussion. The combination of whole-class, small-group, and independent or partner work ensures that all students actively participate.

Richmond does well in facilitating the discussion so that students build on each other's observations and insights. During the discussion on graphs, Richmond has each group present its graph and then encourages other students to ask questions [at 00:37:52:00]. Richmond has created an atmosphere of safety and trust in the classroom. Students freely share their thinking and are respected by both Richmond and their classmates, even when they make a mistake. Throughout the lesson, discussions are open and collaborative.

Indicator B:

While the class session is filled with activities (not just worksheets), no data set or function rule is presented in a real-world context. Only when a student mentions seeing a similar graph on the GED test does Richmond connect the concept to a practical experience [at 00:30:55:00]. Students are only learning the basics of graphing. It's reasonable to wait until future lessons to ask them to apply their knowledge to a real-world or practical problem. The lesson plan mentions that such a problem will be covered in the next lesson. However Richmond does not say so explicitly during his lesson preview.

Indicator C:
When students work with the activities and problems, they are engaged, persistent, and supportive of each other's learning. Richmond also circulates [at 00:02:22:00] and asks questions to help move students' thinking along so they don't get stuck on one concept or idea. During class discussions, students often ask thoughtful questions that show their engagement [at 00:08:52:00], and the small-group presentations of graphs ensure full student participation [at 00:37:42:00].

Core Action 4. Lesson content is intentionally sequenced to develop students' skills and knowledge.	Y, N, or N/A
A. Instructor explicitly relates new mathematical concepts to previous lessons or students' prior knowledge.	Y
B. Instructor delivers concepts in a way that builds on their logical connections to each other.	Y
C. Instructor ends the class by: • Reviewing lesson objectives; • Summarizing student learning with references to student work and discussion; and • Previewing the next class session and explaining how it will build on today's activities.	Y

Evidence observed:

Indicator A:

Richmond begins with a warm-up to clarify students' understanding of functions [at 00:00:25:00]. He reviews a function's definition by having students identify a rule based on the first few entries of a table. He then has them complete the table. He also reiterates the meaning of terms like "rule," "input," and "output," which are key terms to writing statements or equations to represent functions [at 00:05:32:00].

Indicator B:

The lesson focuses on the three views of a function: a rule, table, and graph. The concepts build and flow smoothly. Richmond first reviews student understanding of finding a table of solutions based on the rule [at 00:00:25:00]. He moves on to creating a graph [at 00:22:41:00]. He then gives students a graph and asks them to create a rule and a table [at 01:01:46:00]. By the end of the lesson, when he gives students a rule, table, or graph, most can create the other two views of that function. He also uses the input and output calculations to incorporate a conversation on decimals and fractions [at 00:07:32:00].

Indicator C:

At the end of the lesson, Richmond previews the following class [at 1:10:31:00]. Earlier on, he also reviews student learning about the three views of functions: an input/output table, rule, and graph [at 00:28:21:00]. While he doesn't summarize student learning at the end of the lesson, he does so at different points throughout, such as on graphing functions [at 00:59:41:00]. Periodically, Richmond makes connections to upcoming lessons. He talks about looking at different types of equations (nonlinear) in upcoming lessons [at 00:18:10:00]. He also explains that they will be looking at different types of graphs in the coming weeks [at 00:30:31:00].

Core Action 5. Students' levels of understanding are checked throughout the lesson, and instruction is adjusted accordingly.	Y, N, or N/A
A. Instructor consistently uses informal yet deliberate methods to provide students with prompt, specific feedback to correct misunderstandings and reinforce learning.	Y
B. Instructor consistently provides strategic supports and scaffolds to students who need them.	Y
C. Instructor provides opportunities for students to evaluate and reflect on their own learning.	N

Evidence observed:

Indicator A:

Richmond checks for student understanding in numerous ways:

- Any time students work in groups, Richmond circulates, checks their work, and asks questions to confirm their understanding.
- After students complete the graphing activity, each group presents its work, allowing Richmond to gauge everyone's understanding of the content. Richmond stands back and looks, then asks, "How did we get different graphs?" [at 00:38:07:00]. Students then offer theories. Richmond continues to ask questions until the class realizes that different scales change the look but not the substance of a graph.
- He also calls on different students during whole-class discussions.
- When students are confused about determining an output from a table, Richmond repeats the question rather than simply correcting the wrong answer. He also waits for students to respond or ask questions themselves [at 00:25:47:00].

Richmond provides specific feedback to students throughout the lesson to correct misunderstandings:

- For example, when a group misunderstands the origin on a graph—writing a zero on both the x- and y-axis—Richmond says it is mathematically incorrect [at 00:52:04:00].
- He gives a practical example to help students understand its location at (0, 0)—although he never uses the word "origin."
- He also asks one group to save its incorrect graph, mistakes and all, to provide a "teachable moment" for all students about common misconceptions [at 00:47:07:00].

Indicator B:

Richmond responds to a student's confusion during a class discussion by taking time to review the concepts until she understands [at 00:55:23:00]. Then, while the class is working, he works one-on-one with another student at the board on a graphing question [at 01:07:40:00]. He also tells two students he "is not done with them yet" as he closes out the lesson [at 01:38:00:00].

Indicator C:
Richmond does not explicitly ask students to reflect on their learning. Several students carefully take notes throughout the lesson, but it's not clear whether they are reflecting on their learning or just taking contemporaneous notes [at 00:00:25:38].

LESSON PLAN FOR MATHEMATICS

Created by Richmond Onokpite
Academy of Hope Adult Public Charter School • District of Columbia

Title of this lesson:

The Three Views of a Function: Rule, Table, and Graph

Brief description of how the lesson is to be used:

After being introduced to the concept of function using a function machine, students will expand their knowledge by graphing functions. Students will work with three ways to represent a function:

- A rule—a verbal description or equation;
- A table—two columns showing the corresponding "inputs" and "outputs"; and
- A graph—with horizontal and vertical axes to represent the inputs and outputs, respectively.

Intended instructional level of the lesson:

Low/Middle Intermediate Level

Suggested time to spend on the lesson:

One 90-minute session

Learning goals of the lesson:

Students will be able to:

- Given a rule, the table, or a graph of a function, create the other two views of a function.
- Identify whether an input/output pair is a solution for a given function.
- Create data tables and interpret information from a table.
- Graph in one quadrant using the traditional x-y grid.

The college and career readiness standards addressing the major work of the level (MWOTL) targeted by this lesson:

(8.F.1) MWOTL (Middle Intermediate): Functions – Developing the concept of function and graphing functions in the coordinate plane and analyzing their graphs.
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

(5.G.1) MWOTL (Low Intermediate): Geometry – Developing understanding of the coordinate plane.

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

The Standards for Mathematical Practice that are central to the goals of the lesson and their connections to the content of the lesson:

Primarily:

MP.8. Look for and express regularity in repeated reasoning.
In this lesson, students are asked to complete several function tables and create the function graph. In doing so, they learn how to recognize the rule for a function, based on the pattern they observe in the inputs and their corresponding outputs.

Secondarily:

MP.3. Construct viable arguments and critique the reasoning of others.
In the group discussion after both the "group walk" activity and the graphing activity, students are asked to present and support their findings, recognize errors, and discuss the reasoning of others.

Prerequisite or foundational content students need to succeed in the lesson:

- To apply the function rule and create a table of values, students need to be fluent in basic operations involving whole numbers and decimals.
- Students need an informal understanding of the concept of function.

Description of how the content of the lesson is related to other content taught at the lesson's level:

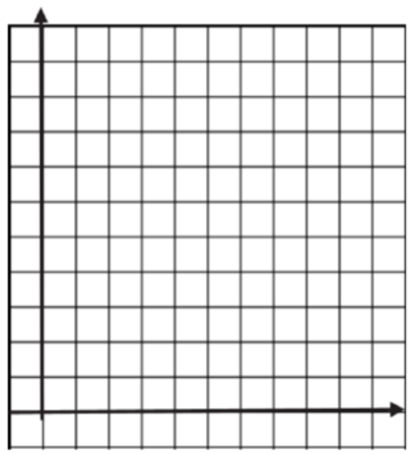
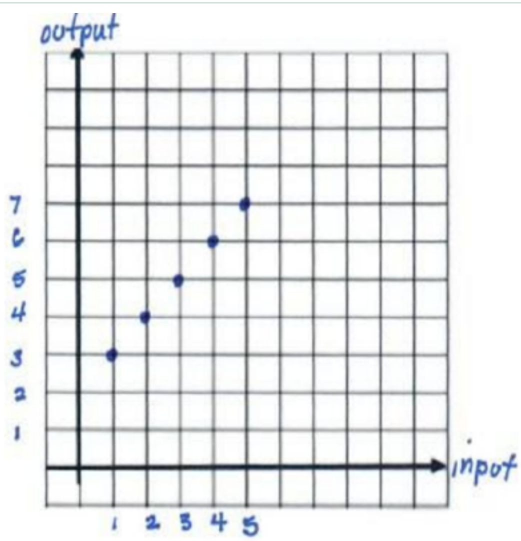
Previously, students reviewed bar graphs, line graphs, circle graphs, and pictographs. They should be familiar with the horizontal and vertical axes and line graphs. They should also know how to apply that knowledge in creating the graph of a function.

In the preceding lesson, students were introduced to the concept of function using a function machine. Students used a one-operation rule to complete a table of values and identified a one-operation rule that corresponds with a table of values.

After completing the current lesson, students will move on to completing a table of values when given a two-operation function rule. They will also identify a two-operation function rule when given a table of values or a graph. This leads to using functions to represent and solve practical problems.

The suggested lesson sequence is as follows:
Engage: Gallery walk [Timestamp: 00:00:20:00]
Students should work in pairs to answer the questions on the charts posted at the “group walk” stations (at tables). They are titled, “What Is My Rule?”; “What Is My Output?”; and “What Is My Input?” This will provide students with several opportunities to describe the rule. Bring the whole group together to discuss the various answers given by the students and to provide students with the opportunity to defend their answers. Students need to be comfortable with finding the function rule and presenting it verbally—before being introduced to the symbolic representation (equation).
Explain: Introduction to the three views of function [Timestamp: 00:10:16:00]
Find a rule and its equation. Once students discover the rule for a set of ordered pairs, ask them to write the “equation” in words: input … rule = output. For example, if the rule is +2, it is: input + 2 = output. Then offer them the option of replacing the words “input” and “output” with letters (variables). In this example, the new equation is a + 2 = b. Make sure each table group has a chance to write an equation and use rules that involve multiplication. For example, if the rule is x6, it is: input x 6 = output. Here, students need to know that by convention it is most appropriate to put numbers before letters for multiplication. 6a = b is more conventional than a x 6 = b or a6 = b.
Build a table. Write numbers 1 through 5 in the input column of the table. Write in “+2” as the rule, and ask the students to complete the outputs. Make a graph. Tell the students that a graph is used to show the values of the inputs and outputs for a function. It is sometimes used instead of a table, or in addition to it. Go through one example of plotting points on a graph for the class. To do this, create a large version of the example graph on the board. Or on large graph paper that can be attached to the board, or project a version of the graph onto a screen.

Rule + 2		
In	Out	
1	3	(1, 3)
2		
3		
4		
5	21	

Make a graph. Use the horizontal axis for the inputs and the vertical axis for the outputs. Plot the input of 1 and the output of 3 on the graph. Explain the process to the students, and then call out different students to plot the other input/output pairs.

Use the word "coordinates" instead of "input/output pairs" or "points." Coordinates come from the input/output in the table and must be written as pairs. Write the first ordered pair in the third column of the function table and have students call out the remaining input/output pairs. At this point, students should say what they notice about the graph and whether or not they can connect all the points. This is a good occasion to ask students if it's possible for the graph to be a zigzag (nonlinear) instead of a straight line (linear). *[Solution: Ordered pairs – (1, 3), (2, 4), (3, 5), (4, 6), (5, 7) Graph – see below]*


Explore: Graph functions [Timestamp: 00:31:20:00]

Make sure the students are comfortable with the procedure before giving them a new rule which they will use to create a table and a graph.

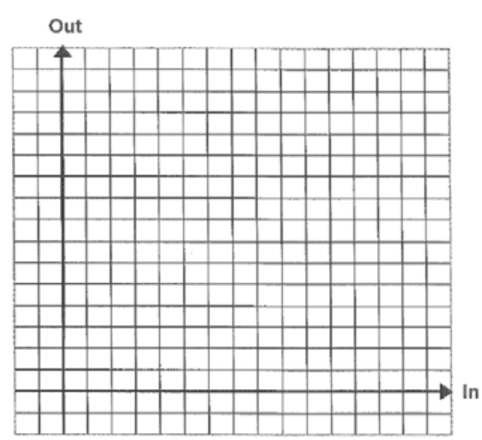
Have students work in pairs or small groups to complete the function tables below (Questions 1 and 2). They should use the rule provided and then create the corresponding graphs by plotting the ordered pairs (input, output) from their tables. Point out that these tables start with 0, rather than 1. Make sure students have a clear understanding of (0, 0) as a position on the coordinate plane (the intersection of the two axes).

Move around the room to observe students' graphs and provide necessary guidance and feedback.

Once students have completed the activity, each group should present its graph to the class. The other students in the class should be given the opportunity to ask questions and critique the graphs. There may be differences in the way the graphs look, based on how students selected and spaced the units for the two axes. This is a good time to discuss the importance of using a uniform scale in creating a graph. [NOTE: In the graphs shown below, the units are not identified. Students should be reminded to identify the scale before they plot any points.]

Question 1:

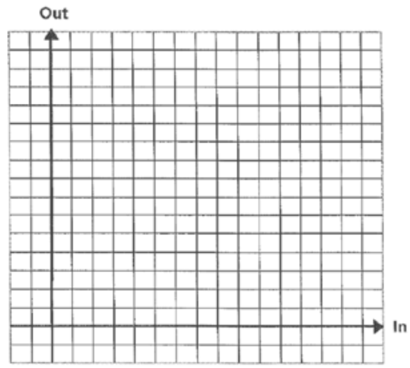
Rule: + 3		
In	Out	Solution
0		
1		
2		
3		
4		
5		
6		
7		



[Solution: Ordered pairs – (0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9), (7, 10)]

Question 2:

Rule: x 2		
In	Out	Solution
0		
1		
2		
3		
4		
5		
6		
7		



[Solution: Ordered pairs – (0, 0), (1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12), (7, 14)]

Extend: Working to create a function table from a graph [Timestamp: 00:59:55:00]

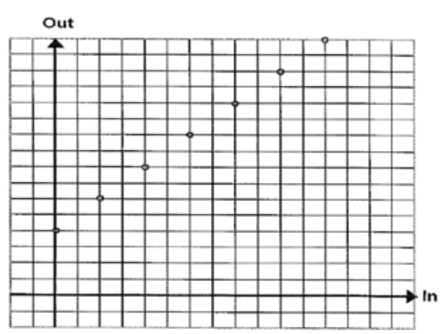
Once students can complete the function tables and draw the graphs with little or no teacher assistance, begin the next activity. Present a blank function table along with a graph that identifies the points as dots only. Students should work in pairs or small groups to come up with the function table and determine the rule. Once students have completed the activity, each group should explain and defend its findings to the class.

[NOTE: In this example, the units are not identified, so the assumption is that each mark represents one unit.]

*[Solution: Ordered pairs – (0, 4), (2, 6), (4, 8), (6, 10), (8, 12), (10, 14), (12, 16)
Equation – output = input + 4, or b = a + 4]*

The "application" rigor component, which gives students an opportunity to think deeply about math and solve real-life problems, will be covered in the next lesson.

Rule:		
In	Out	Solution



MATHEMATICS CLASSROOM OBSERVATION FOR WILLIAM WALKER

What follows is an observation of a lesson taught by William Walker. He served as an instructor from the Academy of Hope Adult Public Charter School in Washington, D.C. This observation provides concrete examples of what challenging state academic standards in mathematics look like in daily planning and practice. The observation tool is designed as a professional development tool for instructors, those who support instructors, and others working to implement standards. It is not designed for use in evaluation.

Core Action 1. Lesson content is rigorous and relevant for the level defined by the state-adopted standards.

Core Action 2. Learning activities are cognitively demanding and maximize opportunities for students to master the lesson content.

Core Action 3. Lesson content productively engages students.

Core Action 4. Lesson content is intentionally sequenced to develop students' skills and knowledge.

Core Action 5. Students' levels of understanding are checked throughout the lesson, and instruction is adjusted accordingly.

Core Action 1. Lesson content is rigorous and relevant for the level defined by the state-adopted standards.	Y, N, or N/A
A. Instructor presents a lesson with well-defined standards-based goals that focuses on the major work of the level (MWOTL).[1]	Y
B. Instructor presents a lesson that addresses the Standards for Mathematical Practice that are central to the lesson goals and connected to the targeted content.	Y
C. Instructor, when addressing the MWOTL, intentionally targets one or more aspects of rigor as appropriate for the addressed standard(s). Mark the aspect(s) of rigor the lesson addresses: • Conceptual understanding • Procedural skill and fluency • Application	Y

Evidence observed:

Indicator A:

All of the CCR standards listed in William's lesson plan are reflected, at least partially, in his lesson:

- Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (4.OA.5)
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. (5.OA.2)
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (8.F.1)

All three standards targeted in this lesson are central to students' learning.

During the lesson, students engage in a variety of tasks and activities that reflect the standards.

[1] MWOTL – Major work of the level (MWOTL) – The most important mathematics in preparing adult learners for success in college, careers, and life. Instructors should learn to identify the topics that are—and are not—major topics for the various standards' levels. When planning instruction, they should focus deeply on the MWOTL rather than racing to cover topics in a mile-wide, inch-deep curriculum. The standards recently adopted by states require educators to significantly narrow and deepen the way time and energy is spent in the math classroom. When educators apply a MWOTL approach to teaching and learning, students can gain strong foundations in critically important areas: *conceptual understanding, procedural skill and fluency, and the ability to apply what they learn to solve math problems inside and outside the classroom.* Students should spend most of their time on the MWOTL; it should be the major focus of instruction. https://lincs.ed.gov/publications/pdf/ccr/Math_Unit_1 Materials/Math_1_part_mat.pdf

William addresses the concept of a function as a rule that assigns each input exactly one output during the lesson. He also encourages students to identify patterns they notice in the table. During the first activity, students are given a rule (two-operations) and must complete a table with some inputs missing for corresponding outputs, and vice versa. Students report out their answers from their completed worksheet [at 00:11:39:00].

William presents the lesson goals to students at the beginning of the video segment. During this introduction, we also see interview footage of him explaining the lesson goals.

Indicator B:

Standards for Mathematical Practice are identified in the lesson plan and reflected in the lesson, specifically, MP.8: Look for and express regularity in repeated reasoning. To elicit these habits of mind, students must complete a function table based on a two-operation rule during the first activity. They earn how to find the output based on the given rule and use it to find the input based on a given output.

Indicator C:

Since this lesson's focus is to introduce and explore two-operation rules for functions, conceptual understanding is the primary goal. This lesson expands on the previous lesson, where students identified an input or output based on a one-operation rule. William asks questions to check student understanding. He first works through a problem with the group and then has students complete the remaining problems with a partner. He then has students report out to make sure they understand.

Core Action 2. Learning activities are cognitively demanding and maximize opportunities for students to master the lesson content.	Y, N, or N/A
A. Instructor presents high-quality questions and tasks to prompt students to discuss their developing thoughts and elaborate on and justify their responses.	Y
B. Instructor consistently uses explanation, modeling, or examples to make the mathematics of the lesson explicit.	Y
C. Instructor provides students with opportunities to work with and practice level-specific problems and exercises.	Y

Evidence observed:

Indicator A:

William makes the initial activity challenging by asking students to find an input from its output; this activity requires students to work backward [at 00:02:36:00]. Moreover, some of the partner activities include decimals, which also can be challenging.

William shows students upfront how to find an output given the two-operation rule. Once many students demonstrate that they have a foundational understanding, the class plays a round of "the function game." This game asks students to identify another ordered pair for an unknown rule when given a single ordered pair [at 00:13:16:00]. Students call out their guesses until William writes several ordered pairs in the table on the board. Eventually, many students show they are able to find the pattern.

Students discuss the problems productively. William often asks probing questions to move students forward in their thinking. For example, when a student answers the question during the initial activity, William encourages him to elaborate on the answer [at 00:02:07:00].

William often encourages students to justify and elaborate on their responses. When students work in small and whole groups, William asks questions rather than just imparting information. His questions require more than a "yes" or "no" answer from students. He does not simply give students the answers; he leads them to the correct answers. He often tries to clarify students' understanding, asking them to explain how they got their answers. In his interview, William describes his intentionality to lead through questions [at 00:06:03:00].

Indicator B:

During the whole-group discussions, William uses explanation, modeling, and examples. When he introduces the concept of finding the input/output based on a two-operation rule, William builds from students' previous knowledge of doing so. After showing an example, he has students work on tables with their partners. He also asks many questions throughout.

Indicator C:

The problems and activities presented are appropriate for Low/Middle Intermediate-level students and match the standards at that level. William presents a variety of problems during the short segment. William works through an example of finding the input/output. He bases this on a two-operation rule with students, and then they complete a few problems on their own and report out. They also play "the function game" [at 00:13:16:00]. All students are participating and are engaged in each of these activities and in-class discussions. Students ask questions, discuss, and volunteer answers throughout the lesson.

Core Action 3. Lesson content productively engages students.	Y, N, or N/A
A. Students participate actively in sustained class discussions and activities where they build on each other's observations and insights.	N/A
B. Students have varied opportunities to apply what they are learning in authentic adult-oriented contexts.	N/A
C. Most students display persistence with tasks and problems.	N/A

Evidence observed:

This is only a video segment of William's lesson (and is not full length). No evidence has been gathered to exemplify the indicators that make up this Core Action.

Core Action 4. Lesson content is intentionally sequenced to develop students' skills and knowledge.	**Y, N, or N/A**
A. Instructor explicitly relates new mathematical concepts to previous lessons or students' prior knowledge.	N/A
B. Instructor delivers concepts in a way that builds on their logical connections to each other.	N/A
C. Instructor ends the class by: • Reviewing lesson objectives; • Summarizing student learning with references to student work and discussion; and • Previewing the next class session and explaining how it will build on today's activities.	N/A

Evidence observed:

This is only a video segment of William's lesson (and is not full length). No evidence has been gathered to exemplify the indicators that make up this Core Action.

Core Action 5. Students' levels of understanding are checked throughout the lesson, and instruction is adjusted accordingly.	**Y, N, or N/A**
A. Instructor consistently uses informal yet deliberate methods to provide students with prompt, specific feedback to correct misunderstandings and reinforce learning.	Y
B. Instructor consistently provides strategic supports and scaffolds to students who need them.	Y
C. Instructor provides opportunities for students to evaluate and reflect on their own learning.	N/A

Evidence observed:

Indicator A:

William checks for student understanding in numerous ways: any time students work with a partner, he circulates, checks their work, and asks questions.

As noted, William more often leads students to the correct answers through questioning than by providing them. When a student tries to check an answer using the opposite operation instead of the original operation, William asks the group questions, clarifying the process. He then leads the student toward correcting his answer [at 00:04:54:00]. During whole-class discussions, William calls on several different students; students are comfortable volunteering.

William provides specific feedback to students throughout the lesson to correct misunderstandings. For example, when a student has trouble completing a whole-group example, he first walks her through that example. Then he walks her through two additional examples using a series of questions [at 00:07:38:00].

Throughout, William allows students plenty of time to express their answers and ideas. For example, when a student starts down the wrong path, William gives him time to express his answer before going back and addressing the misconception [at 00:04:54:00].

Indicator B:

During the whole-group discussion, William responds to a student who has a misunderstanding when "checking" his answer [at 00:04:54:00]. He also provides supplemental instruction to a student who has trouble working independently through the problems [at 00:07:38:00]. In both instances, he works with them until they understand the content.

Indicator C:

William does not explicitly ask students to reflect on their learning as a group. This video is only a segment, however, and does not show the lesson's close.

LESSON PLAN FOR MATHEMATICS

Created by William Walker
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Title of the lesson:

Function Rules: Two-Operation Functions

Brief description of how the lesson is to be used:

In previous lessons, students were introduced to function rules with one operation. In this lesson, students will be introduced to function rules that involve two operations.

Intended instructional level of the lesson:

Low/Middle Intermediate Level

Suggested time to spend on the lesson:

One 90-minute session

Learning goals of the lesson:

Students will be able to:

- Complete a table of values when given a two-operation function rule.
- Identify a two-operation function rule when given a table of values.
- Perform two-operation functions with decimals.

Functions that involve two operations give students opportunities to work with problems in authentic contexts. This also allows them to brush up on arithmetic skills, and to notice patterns of change in tables of values. When completing both sides of an input/output table, students have an opportunity to "think backward." This means inversing the operations used to produce an output in order to find the corresponding input.

The college and career readiness standards addressing the major work of the level (MWOTL) targeted by this lesson:

(4.OA.5) MWOTL (Low Intermediate): Algebra – Writing, evaluating, and interpreting expressions and equations.
Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

(5.OA.2) MWOTL (Low Intermediate): Algebra – Writing, evaluating, and interpreting expressions and equations.
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

(8.F.1) MWOTL (Middle Intermediate): Functions – Developing the concept of function. Graphing functions in the coordinate plane and analyzing their graphs.
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

The Standards for Mathematical Practice that are central to the goals of the lesson and their connections to the content of the lesson:

MP.8. Look for and express regularity in repeated reasoning.
In this lesson, students complete several function tables based on a two-operation rule. They also identify a rule based on a table of values. Identifying patterns when completing the function tables will help them identify the rules based on a table of values.

Prerequisite or foundational content students need to succeed in the lesson:

- Students need familiarity with input/output tables.
- Students need fluency with operations and number sense.
- Students need to understand the concept of function and to know how to use one-operation rules.

Description of how the content of the lesson is related to other content taught at the lesson's level:

Previously, students were introduced to the concept of function using a function machine.

Students used a one-operation rule to complete a table of values. They were able to identify a one-operation rule that corresponds with a table of values.
In the preceding lesson, students worked with three ways to represent a function:

- A rule (a verbal description or equation).
- A table (two columns showing the corresponding "inputs" and "outputs").
- A graph (with horizontal and vertical axes to represent the inputs and outputs, respectively).

After completing the current lesson, "Function Rules: Two-Operation Functions," students will apply the content addressed in this lesson.

The suggested lesson sequence is as follows:

Show students how to "undo" the operations, working backward. When going through the example, don't just tell students the input/output numbers; guide students in working through the problem.

Work with the whole group on the first function from the handout (shown below). It is appropriate for students to use trial and error, but ask them to make a reasoned first guess.

Rule: x2 then +1	
In	Out
2	
4	
6	
8	
	21
	31

To encourage students to think along these lines, ask them the following questions:

- Do you think the input that produces 21 will be bigger or smaller than the other inputs? Why? *[Answer: Bigger, because the outputs for 2 through 8 are getting bigger.]*
- What is the last thing we do according to the rule? *[Answer: Add 1.]*
- So, what was the number in this case just before we added 1? *[Answer: 20.]*
- Good. Now, what input will get me to 20 before adding 1? *[Answer: 10.]*

One of the most important activities in the lesson comes next, so take the necessary time for students to work through it. Before distributing the handout, ask students to figure out the two-operation rule from a table of values (like the one below). Work as a whole group on this activity.

Rule:	
In	Out
1	5
2	7
3	9
4	11
5	13
6	15

When I go from the input to the output, how do the numbers change? *[Answer: They get bigger.]*
What operations often make numbers bigger? *[Answer: Addition and multiplication.]*
What's one way I can get from 1 to 5? *[Answer: Add 4.]*
Is that my function rule? Why or why not? *[Answer: No, because 2 plus 4 does not equal 7.]*
What else can we do to get from 1 to 5? *[Answer: Multiply by 5.]*
Is that our function rule? Why or why not? *[Answer: No, because 2 times 5 does not equal 7.]*
OK, it can't work with simple addition or multiplication. We are going to need to do two operations. How can we use multiplication and then addition to get from 1 to 5? *[Answer: Multiply by 4 and then add 1.]*
Does that work for the rest of the inputs/outputs? Why or why not? *[Answer: No—it doesn't work for 2 (2 x 4 = 8, 8 + 1 = 9; I want 2 to produce 7).]*

[Solution: input x 2 and then + 3 = output, or times 2 and then add 3]

Reminders:
- It is good for students to spend some time using trial and error to figure out the function rules. Don't jump in to offer shortcuts.
- Be on the lookout for tables that can be described by more than one rule.
- If students notice there is a constant change in the outputs when the inputs are consecutive, praise them and ask if there's a pattern. This will be explored more deeply in subsequent lessons.

Now it is time to play the "function game." Create a one- or two-operation function rule using multiplication and/or addition (like the solution for the table below). Complete a few inputs and outputs. Ask students to offer an input and output based on what they think the rule is. Tell students if their pairs are correct or not, but do not tell them the rule. The goal is for students to guess the rule.

Rule:	
In	Out
5	12
6	14

[Solution: input x 2 and then + 2 = output, or multiply by 2 and then add 2]