A Little Bit About You – Poll Question

On a scale of 1–5, how familiar are you with the College and Career Readiness Mathematical Content and Practice Standards for Adult Education?

- 1 (no experience)
- 2
- 3
- 4
- 5 (very experienced)

Standards-in-Action 2.0

Cultivating a Language and Content Focus for English Learners

Mathematics Lesson: Ratios and Proportions

Disclaimer

WELCOME BACK!

Shared Norms

1. Be present and engage fully in all activities.

2. Ask questions through the chat or by raising your hand.

3. Put cameras on whenever possible.

4. Prepare for productive struggle.

5. Respectfully challenge one another and withhold judgments for differing perspectives or learning styles.

Session Overview

- Introduction to the Model Mathematics Lesson

- Phases 1–2 of the Model Lesson

  - Break: 60 minutes

- Phases 3–5 of the Model Lesson

- Wrap-Up & Preview of Next Session

Participant Materials – Table of Contents

- Section 1: Notes

- Section 2: Directions for Team Time

- Section 3: Worksheets

- Section 4: Summary of Strategies, Routines, and Teacher Moves

INTRODUCTION TO THE MODEL LESSON DESIGNED FOR ENGLISH LEARNERS (40 MINUTES)

Teaching and Learning Mathematics

| FROM	TO
Organizing learning as a series of stimulus-response interactions.	Creating opportunities for English learners to learn through a series of social and academic interactions.
Teachers as transmitters of information.	Building a community of learners to process, understand, and interpret content.
Developing language in isolation.	Promoting the simultaneous building (and production) of language and disciplinary practices and knowledge.

Intersections Between Language and Content in Mathematical Lessons

Educators need to create opportunities for learners to see and understand that content and language development are interconnected:

1. Language, more than a communication tool, is a thinking tool.

2. More than learning "academic language," they need to learn to use language for academic purposes.

3. Mathematical discussions lead to co-construction of meaning.

4. Content and language development are interconnected.

Let's Explore What Mathematical Proficiency Means to You
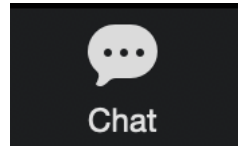
CHAT: Please share your thoughts about:

➢ What it means to be proficient at "doing math."

  – Speak to the connections between and among procedural fluency, conceptual understanding, and strategic competence.

➢ How these three aspects of doing math differ from one another.

Mathematical Proficiency

Conceptual Understanding

Strategic Competence

Productive Disposition

Adaptive Reasoning

Procedural Fluency
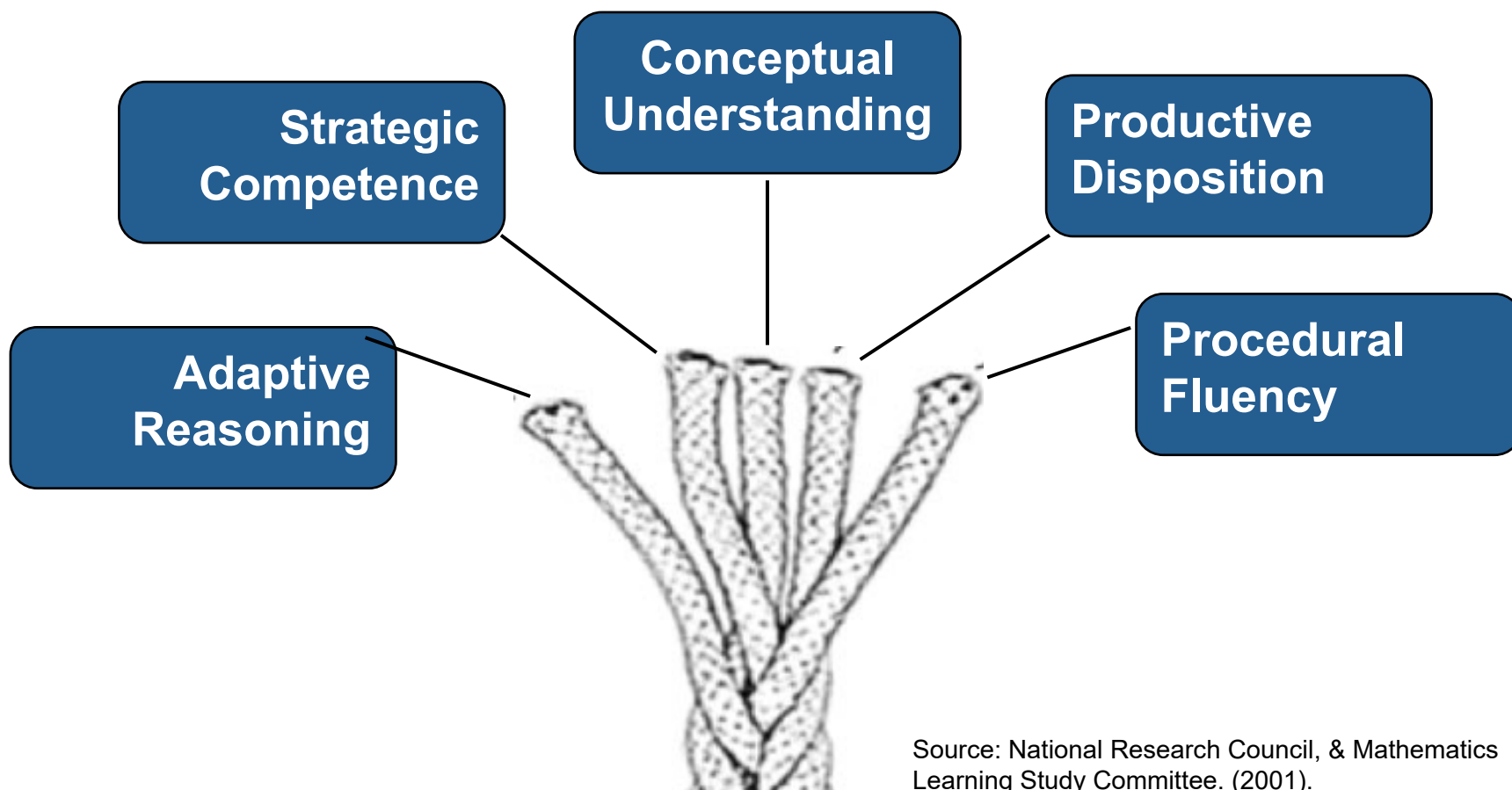
Source: National Research Council, & Mathematics Learning Study Committee. (2001).

Diagnostic Teaching: A Model Lesson
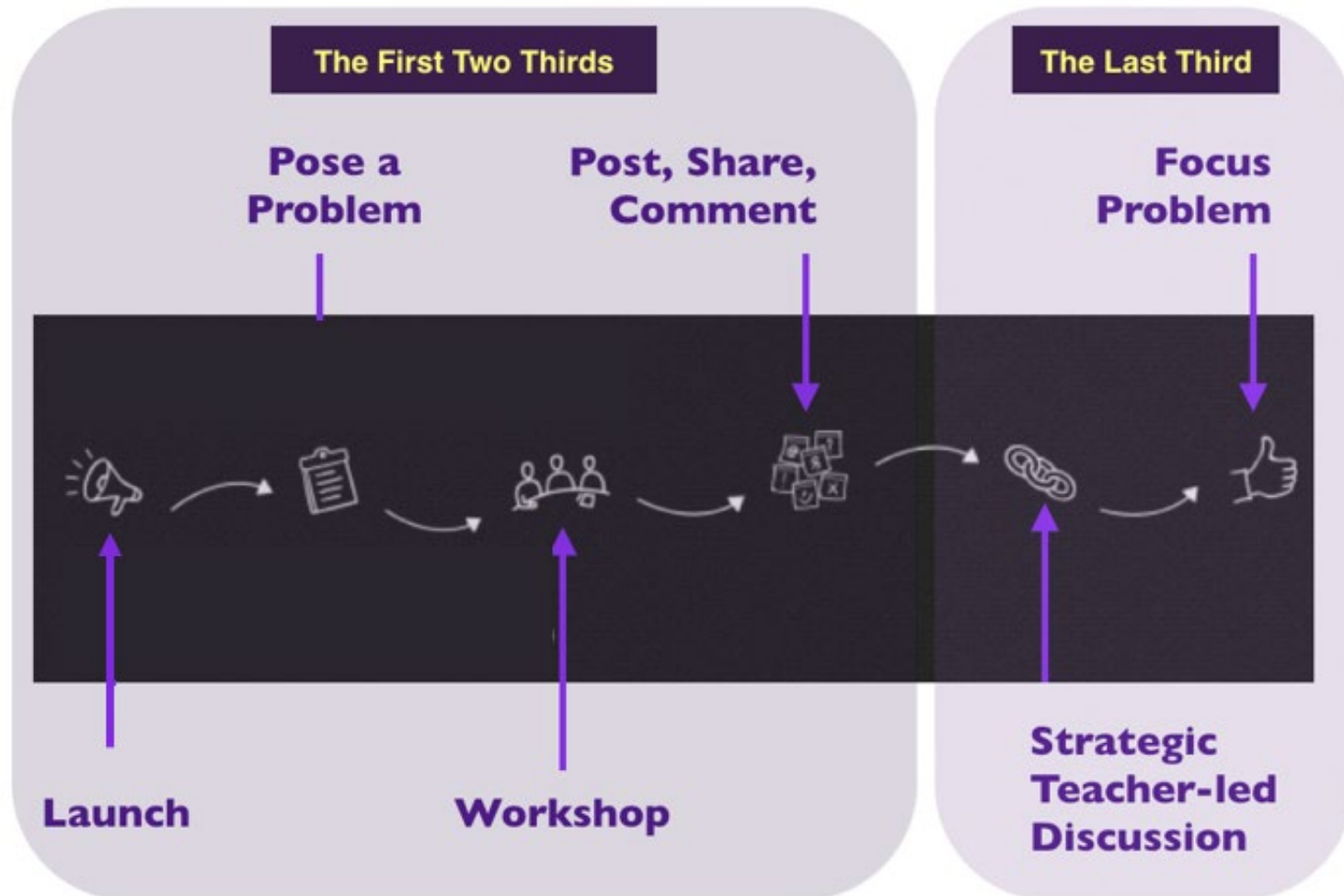
What Is Diagnostic Teaching?

Diagnostic teaching starts with what students know, using formative assessment and building from there:

- The *first two thirds* of the lesson promote divergent thinking by emphasizing students talking and listening to one another through various mathematical language routines (MLRs).

- The *last third* promotes convergent thinking to reach a specific way of thinking and understanding.

Teacher Moves

1. Just-in-time, not just-in-case supports.

2. Students are the audience for students.

3. "Everyone ready?"

4. Use "make an expert" then "turn and talk."

5. Use direct instruction near the END of the lesson.

Team Time: 10 minutes

1. Discuss how you think these **socio-mathematical norms** promote learning:

   - Errors are gifts; they promote discussion and learning.

   - The answer is as important as the mathematical thinking.

   - Ask questions until ideas make sense.

   - Think with language and use language to think.

   - Use multi-modal communication.

2. Be prepared to share comments or questions in the chat when we reconvene as a whole group. (We will call on select teams to share verbally.)

Whole Group Reflection: 5 minutes
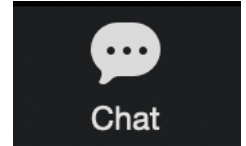
Chat

CHAT: Write your comments:

➢ On the socio-mathematical norms:

- Errors are gifts; they promote discussion and learning.

- The answer is as important as the mathematical thinking.

- Ask questions until ideas make sense.

- Think with language and use language to think.

- Use multi-modal communication.

➢ Any questions?

The Model Lesson Is Designed to Provide. . .

. . .every English learner with the opportunity to *wrestle with*, *make sense of*, and *communicate* about relevant mathematics!

The Model Lesson Includes Mathematical Language Routines

Mathematical Language Routines (MLRs) are designed to:

1. Support sense-making by amplifying rather than simplifying language;

2. Optimize student output as they write or speak to help them get progressively better at expressing their thinking;

3. Cultivate conversation to share ideas, fill in knowledge gaps, make and correct mistakes in low-stakes ways;

4. Maximize linguistic and cognitive meta-awareness.

https://ell.stanford.edu/sites/default/files/u6232/ULSCALE_ToA_Principles_MLRs__Final_v2.0_030217.pdf
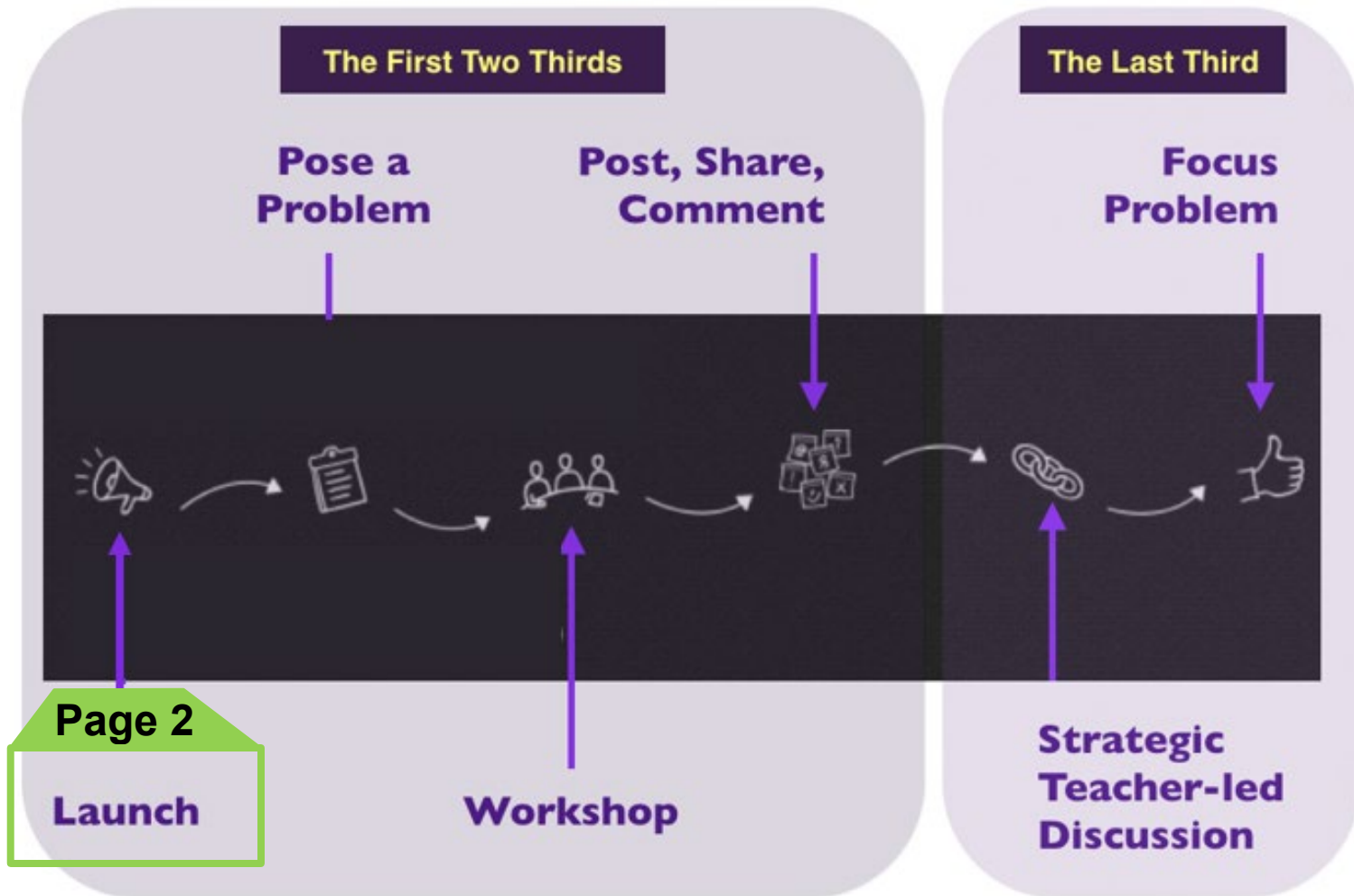
Team Time (MLR Jigsaw): 20 minutes

1. Divide your team into four subgroups. Each subgroup will read two MLRs (pages 23–25). Note what the MLR is and why it is useful and then be ready to share. (5 minutes)

   - Subgroup 1 – MLR 1 and MLR 2

   - Subgroup 2 – MLR 3 and MLR 4

   - Subgroup 3 – MLR 5 and MLR 6

   - Subgroup 4 – MLR 7 and MLR 8

2. Share within your groups. (15 minutes)

PHASE 1: LAUNCH THE LESSON (55 MINUTES)

Diagnostic Teaching: Phase 1

The First Two Thirds

Pose a Problem

Post, Share, Comment

The Last Third

Focus Problem

Page 2

Launch

Workshop

Strategic Teacher-led Discussion

Taking Notes

PHASE 1: LAUNCH THE LESSON

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

Page 2 of the Participant Materials

Lesson Content: College and Career Readiness Standards for Mathematics

- Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities. (6.RP.1, Level C)

- Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.3, Level D)

- MP1: Make sense of problems and persevere in solving them.

- MP2: Reason abstractly and quantitatively.

- MP3: Construct viable arguments and critique the reasoning of others.

CCR & ELP Targets in the Model Lesson

CCR Standard for Mathematical Practice	ELP Level 3 Standards
MP1: Make sense of problems and persevere in solving them.	ELP Standard 1. Construct meaning from informational texts. ELP Standard 2: Students participate in oral and written exchanges of information and respond to peer, audience, or reader comments and questions. ELP Standard 3: Students speak and write level-appropriate content.

CCR & ELP Targets in Model Lesson, cont'd.

| CCR Standard for Mathematical Practice	ELP Level 3 Standards
MP1: Make sense of problems and persevere in solving them.	ELP Standard 6: Students analyze and critique the arguments of others. ELP Standard 7: Students adapt their language choices in accordance with the task and using a range of academic and context-specific words. ELP Standard 9: Students create clear and coherent speech and text to develop an informational topic.

CCR & ELP Targets in Model Lesson, cont'd.

CCR Standard for Mathematical Practice	ELP Level 3 Standards
MP3: Construct viable arguments and critique the reasoning of others.	ELP Standard 4: Students construct level-appropriate oral and written claims. ELP Standard 6: Students analyze and critique the arguments of others. ELP Standard 9: Students create clear and coherent speech and text to develop an informational topic.

Answer These Questions

CHAT: Share your answers to these questions:

1. Have you ever made chocolate milk at home?

2. If so, how did you do it?
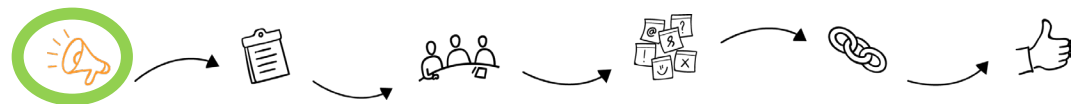
Watch the Video and Ask Yourself:
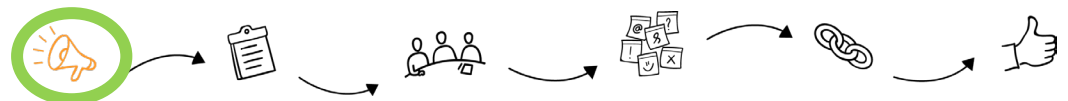

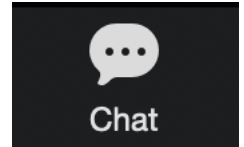**milk** + **chocolate syrup** = **chocolate milk**

(1) How are these recipes the same?

(2) How are they different?

(3) Which is better?

Video on Making Chocolate Milk

Whole Group Reflection: 5 minutes

CHAT: Share your answers to the following questions. (We'll take them one at a time.)

1. How are these recipes the same?

2. How are they different?

3. Which is better?

Team Time: 25 minutes

1. Individually, write down possible mathematical questions that might be asked about the situation, any missing information, or assumptions that are important. (2 minutes)

2. Review your teams' questions, with some brief discussion in team time. (8 minutes)

3. Agree on what questions to copy and select one person to add the questions to the Padlet. (5 minutes)

4. Debrief: Look across at the questions from the other teams in the Padlet. (10 minutes)

Using Padlets

1. At the bottom right-hand corner of your screen, click on the plus sign. This will allow you to post something.

2. A post will come up where you can type your thoughts. The other icons give you the following options: upload, link, search, and snap.

3. Clicking on the three dots to the the right will give you the options above along with others.
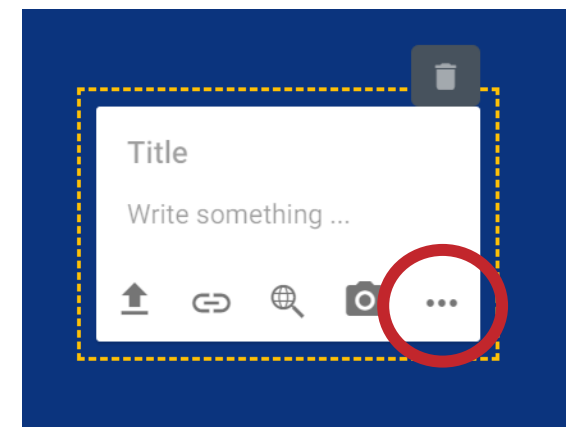
Whole Group Reflection: 10 mins
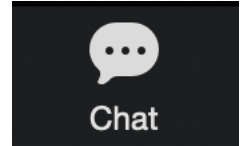
Chat

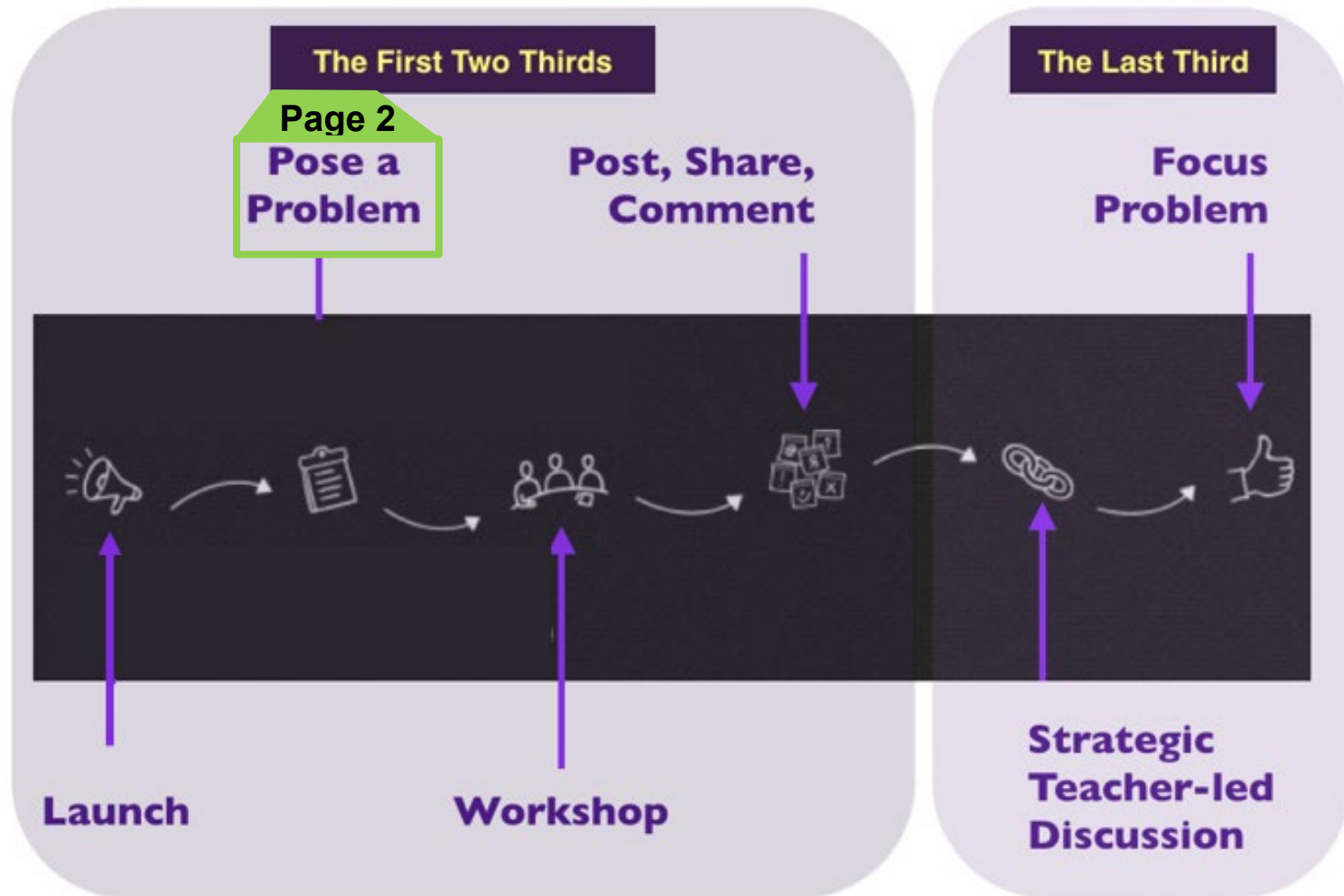CHAT: Share any of your reflections on how MLR "Co-Crafting Questions and Problems":

➢ Allowed you to get inside of a context before feeling pressure to produce answers?
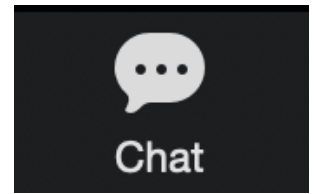
We will give you a minute to jot down notes in your Participant Materials about Phase 1 (page 2).

➢ What Mathematical Language Routines were modeled?

➢ What teacher moves and/or norms were used?

PHASE 2: POSE A PROBLEM (40 MINUTES)

Diagnostic Teaching: Phase 2

The First Two Thirds

Page 2
Pose a Problem

Post, Share, Comment

The Last Third

Focus Problem

Launch

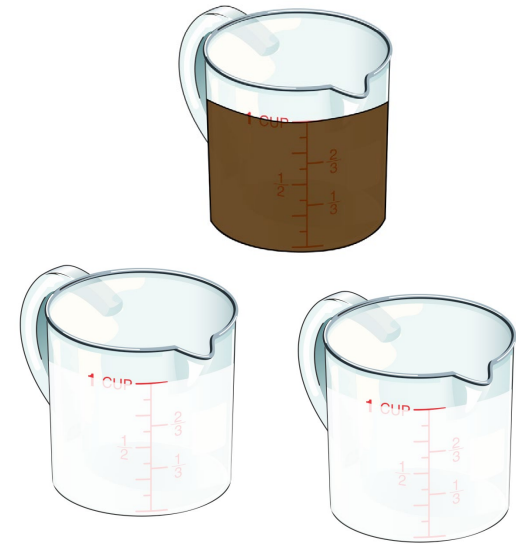Workshop

Strategic Teacher-led Discussion

Answer These Questions

CHAT: Look at Steven's recipe below and answer:
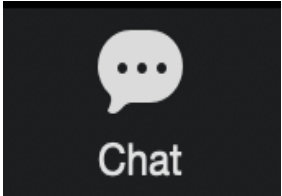
➢ What do you notice?

➢ What do you wonder?

Steven's Award-Winning Mix:

- 1 part Chocolate Syrup
- 2 parts Ice-Cold Milk

[1–2 minutes]

Answer These Questions

CHAT: Share your answers to these questions:

➤ What are the parts in Steven's recipe?

➤ What is the whole?

Chocolate Milk ☐ **part** ☐ **whole**

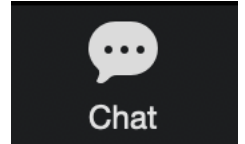**Milk** ☐ **part** ☐ **whole**

Syrup ☐ **part** ☐ **whole**

Poll Question

Does Steven's recipe tell you exactly how much (quantities of) chocolate syrup and milk to use?
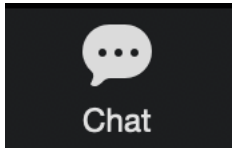
1. Yes
2. No

Could you use Steven's recipe to make a single glass of chocolate milk?

1. Yes
2. No

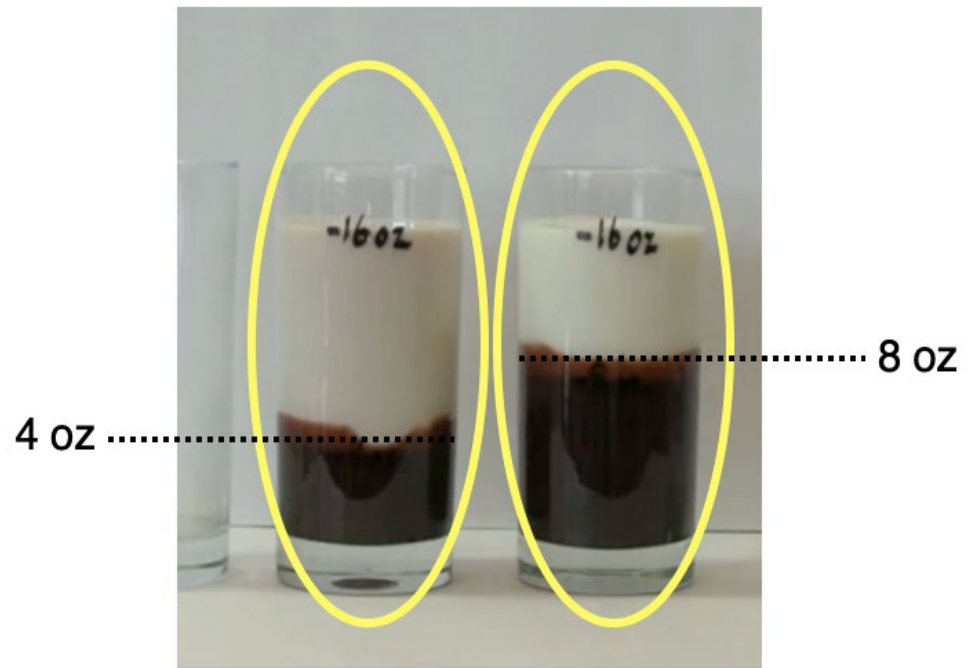Could you use this recipe to make enough for a large team of people?

1. Yes
2. No

How Does Steven's Recipe Compare?

CHAT: Answer whether you think:

➤ Steven's recipe is more intense, less intense, or the same as the two recipes from the video?

Team Time: 20 minutes

1. Individually, fill in the missing values (in the worksheet).
    Go to Worksheet 1 in the Participant Materials (page 16).
    (5 minutes)

2. In Team Time: (15 minutes)

    - Agree on the values you have created.

    - Discuss three questions:

        o Which parts of this table always stayed the same?

        o Why do they stay the same?

        o Which parts changed and why?

3. Be prepared to share the answers to Worksheet 1 and the questions when we reconvene as a whole group.

Worksheet 1 (Page 16)

Student Name: _____

The Intensity of Chocolate Milk

Steven's Recipe – Worksheet ①

Steven's Award Winning Chocolate Milk Recipe

Instructions

Steven made chocolate milk three times this week. He always followed his recipe, but changed the amount of chocolate milk. The first time he used 4 ounces of syrup. The second time he used 1 quart of milk. The third time (for the party) he made a total of 6 gallons of chocolate milk. Complete the table for the missing parts.
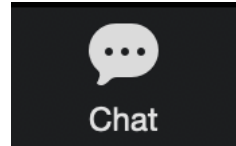
Mix:
1 part chocolate syrup 2 parts cold milk

Steven's Recipe

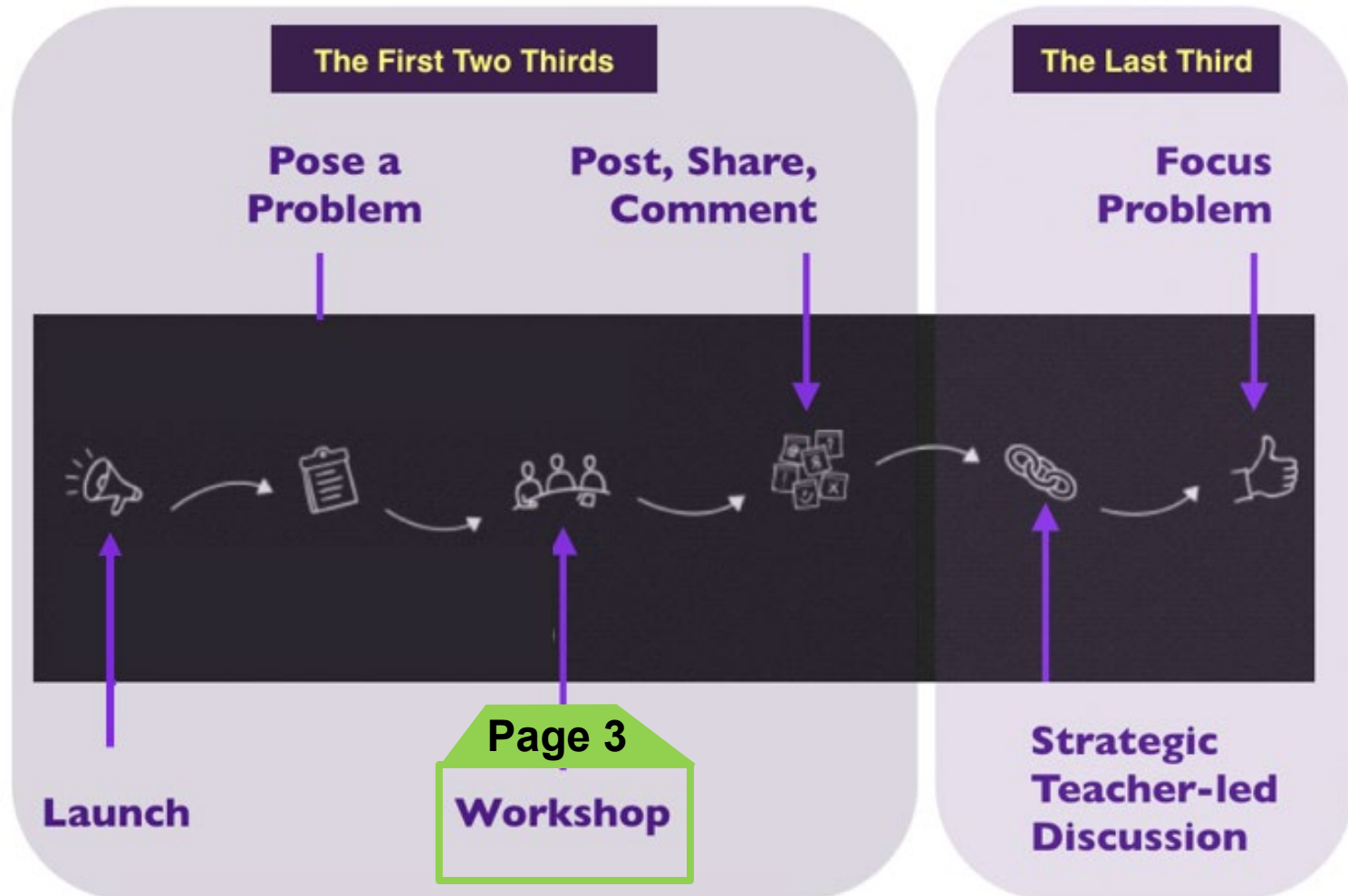|	Chocolate Syrup	Milk	Chocolate Milk	Ratio Syrup: Milk	Ratio Syrup: Chocolate Milk	Ratio Milk: Chocolate Milk
1	4 oz					
2		1 quart				
3			6 gallons			

Whole Group Reflection: 10 minutes

- Let's have a couple of teams share their values and answers to these questions on the worksheet.

- CHAT: Answer the following question.

  ➢ What parts of this table always stayed the same, which parts changed, and why?

- We'll give you a minute to jot down some notes about Phase 2 in your Participant Materials.

  ➢ What Mathematical Language Routines were modeled?

  ➢ What teacher moves and/or norms were used?

BREAK TIME (60 MINUTES)

PHASE 3: WORKSHOP (40 MINUTES)

Diagnostic Teaching: Phase 3

The First Two Thirds

The Last Third

Pose a Problem

Post, Share, Comment

Focus Problem

Launch

Page 3

Workshop

Strategic Teacher-led Discussion

Team Time: 40 minutes

- Your team will create a new recipe for chocolate milk different from Steven's. You choose the intensity.

- You will collaborate on a Jamboard to describe how much chocolate syrup and how much milk to use for three different situations.

- For Situation A and Situation B, use the same recipe. Modify your recipe for Situation C for someone who likes less-intense chocolate milk.

- Make a poster of your new recipe to share with the whole group when we reconvene.

- Worksheet 2 is on page 17 of the Participant Materials.

Using the Jamboard

1. On the top of your screen, you will see different boards. Make sure you click on the board that corresponds to your team.

2. If you hover over the icons on the left, you will see different functions.

1 / 2

Pen

Erase

Select

Add sticky note

Image (add)

Circle

Text

Laser
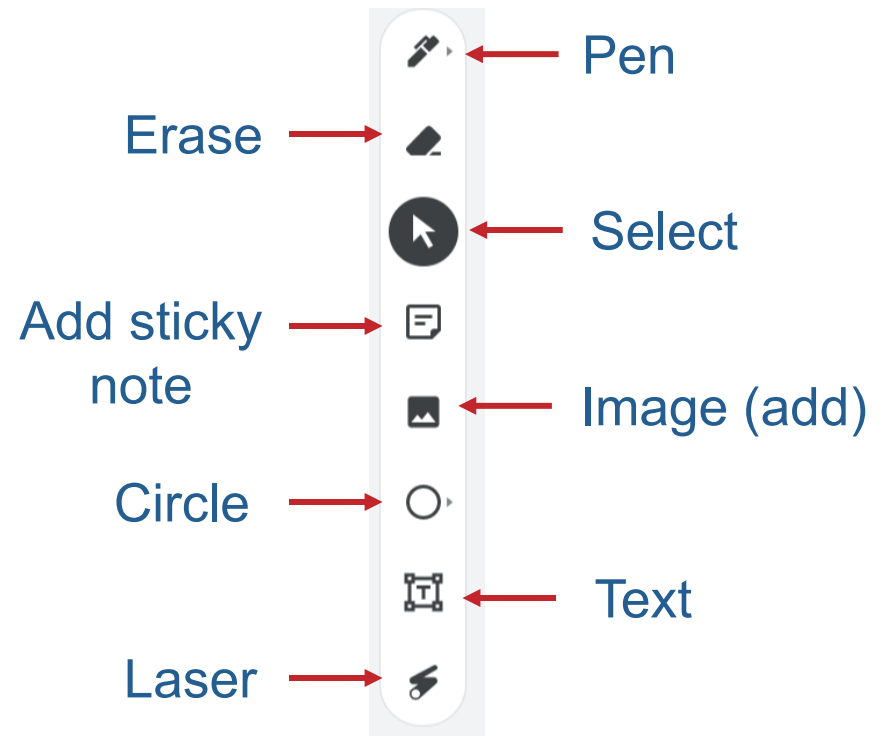
Screenshot of Jamboard

Directions

- Make a poster that your classmates can understand.
- Create a new recipe for chocolate milk different from Steven's. You choose the intensity.
- Use the same recipe for Situation A and Situation B. Modify your recipe for Situation C for someone who likes less-intense chocolate milk.
- Include multiple representations (e.g., drawings, diagrams, tables, equations, graphs) to show how you used your recipe in each situation.
- Include both the ratio of chocolate syrup and milk, and the number of ounces of each ingredient for your recipe.

Chocolate milk for just yourself.
Use your recipe to make one 8-ounce glass of chocolate milk.

Situation A

Chocolate milk for a group.
Use your recipe to make four 6-ounce glasses of chocolate milk.

Situation B

Chocolate milk for your grandma.
She likes very weak (not intense) chocolate milk. Change your recipe to make an 8-ounce glass of chocolate milk for your grandma.
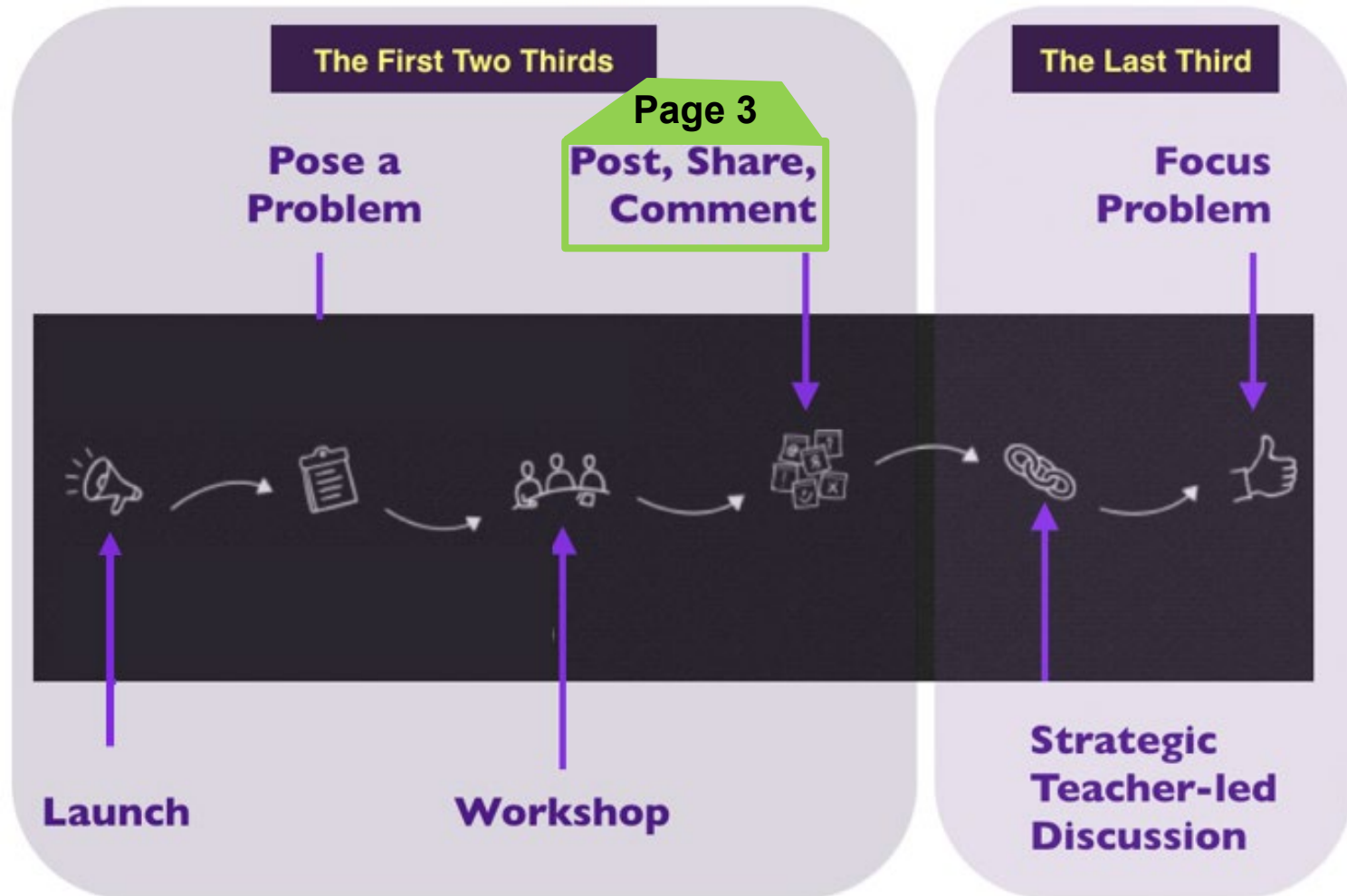
Situation C

Add your names here: First Last, First Last, First Last, etc.

PHASE 4: POST, SHARE, COMMENT (30 MINUTES)

Diagnostic Teaching: Phase 4

The First Two Thirds

Pose a Problem

Page 3
Post, Share, Comment

Launch

Workshop

The Last Third

Focus Problem
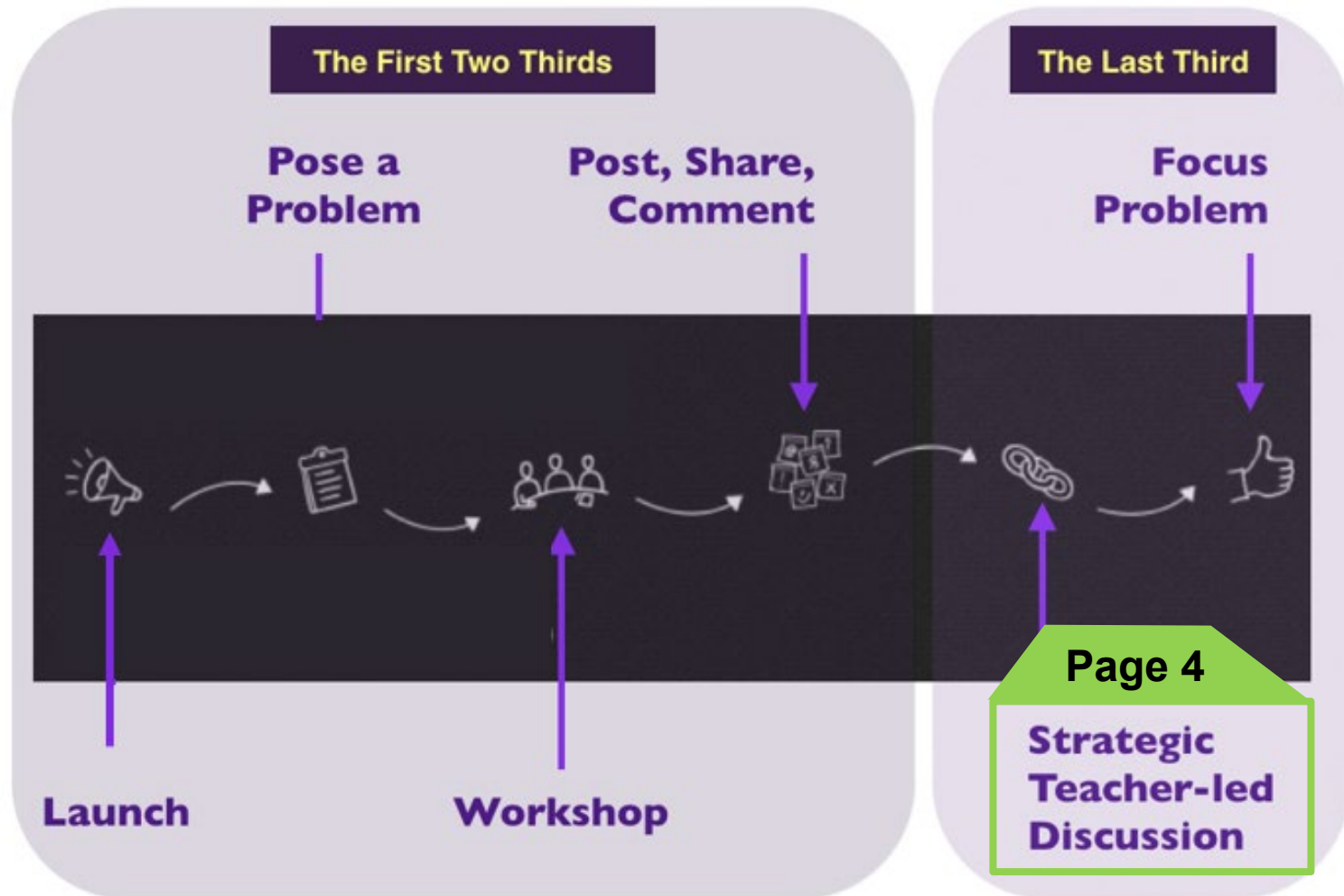
Strategic Teacher-led Discussion

Whole Group Reflection: 15 minutes

- Review one another's "posters" in Jamboard.

- Use virtual sticky notes to ask questions of the other teams' Jamboards.

- We'll give you a minute to jot down some notes about Phases 3 & 4 in your Participant Materials.

  ➢ What Mathematical Language Routines were modeled?

  ➢ What teacher moves and/or norms were used?

PHASE 5: STRATEGIC TEACHER-LED DISCUSSION (40 MINUTES)

Diagnostic Teaching: Phase 5

The First Two Thirds

The Last Third

Pose a Problem

Post, Share, Comment

Focus Problem

Launch

Workshop

Page 4

Strategic Teacher-led Discussion

Phase 5: Strategic Teacher-led Discussion

Let's look at some of the Jamboards — both what worked and what didn't.

Wrap-Up & Preview of Next Session

- In the chat, ask any remaining questions you may have.

- We will focus on the last phase (Phase 6) of the Model Lesson.

WELCOME BACK!
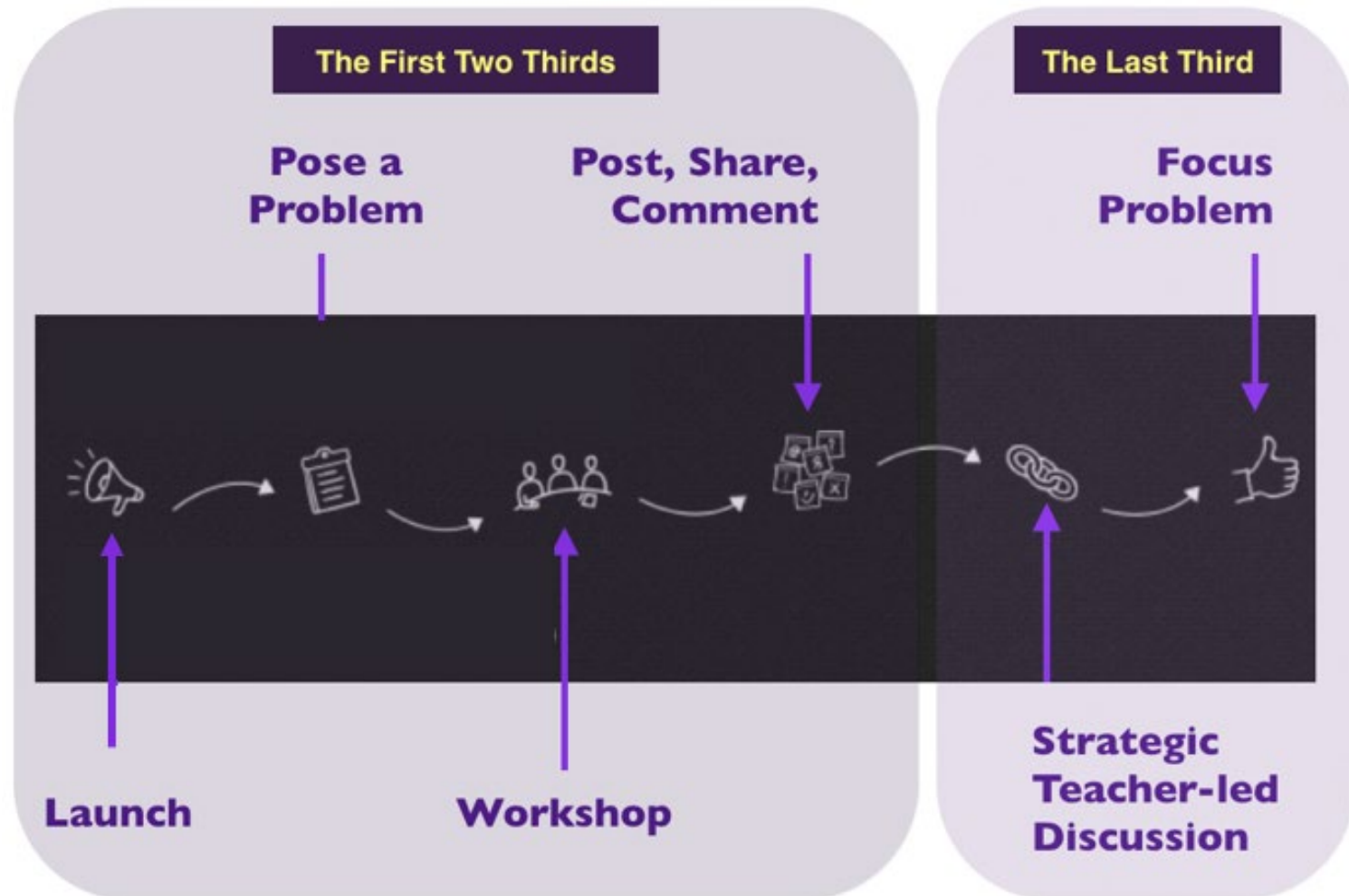
Session Overview

- Phase 6 of the Model Lesson

- Debriefing the Model Lesson and Its Design

- Implications for Your Practice
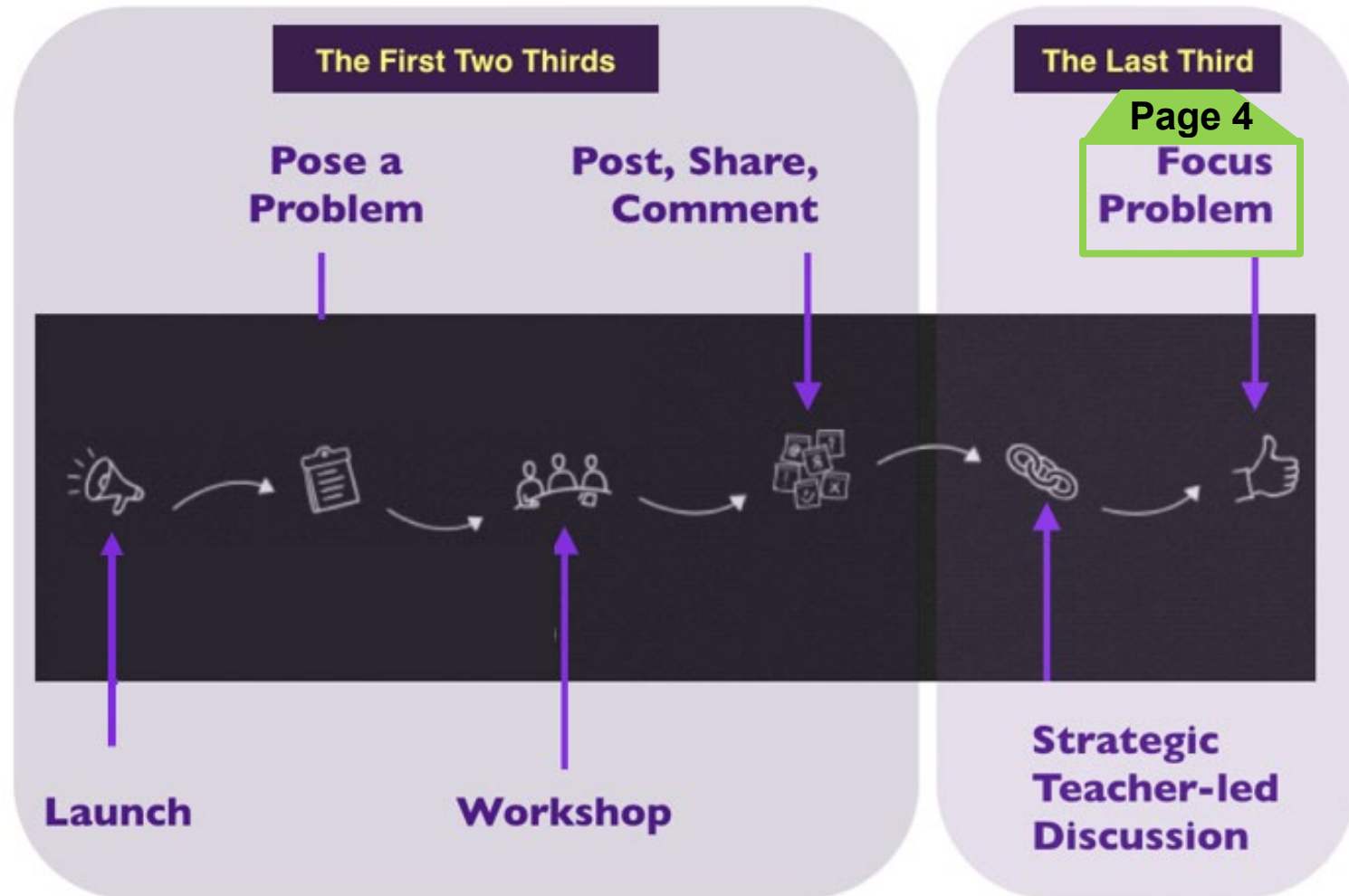
- Wrap-Up

Review of the Session: 20 minutes

- Diagnostic Teaching Phases 1–5

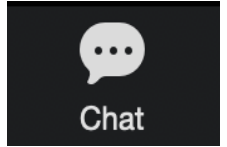- Recap of the Strategic Teacher-led Discussion

Diagnostic Teaching: Six Phases

PHASE 6: FOCUS PROBLEM (40 MINUTES)

Diagnostic Teaching: Phase 6

Chat

Felicia made the winning fruit salad at the Seesaw County Fair. In her recipe, she used 4 pounds of pears, 3 pounds of grapes, 6 pounds of pineapples, and 3 pounds of oranges.

- After First Read, answer in the CHAT: *What is this situation about?* (2–3 minutes)

- After Second Read, answer in the CHAT: *What are all the quantities in this situation?* (2–3 minutes)

- After Third Read, answer in the CHAT: *What are all the mathematical questions we could ask about this situation?* (2–3 minutes)

Team Time: 20 minutes

1. Individually, work on your answers to Worksheet 3 (page 18). (5 minutes)

2. Discuss the answers and make sure the group agrees on the answers. (15 minutes)

3. Select a volunteer to share your answers to the questions—if called upon—when we reconvene as a whole group. (We will call on a selection of teams to share.)

The Intensity of Chocolate Milk

Student Name: _____

From Chocolate Milk to Fruit Salad – Worksheet 3

The Seesaw County Fair: Award-Winning Fruit Salad

Instructions

Felicia made the winning fruit salad at the Seesaw County Fair. In her recipe, she used 4 pounds of pears, 3 pounds of grapes, 6 pounds of pineapples, and 3 pounds of oranges. Answer the following questions using Felicia's winning fruit salad recipe.

1. Based on this recipe, what is the ratio of the weights of oranges to pears?

2. Based on this recipe, what is the ratio of the weights of pineapples to grapes?

Whole Group Sharing: 5 minutes
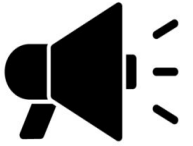
Let's have a couple of teams share their answers to the questions in Worksheet 3:

1. Based on this recipe, what is the ratio of the weights of oranges to pears?

2. Based on this recipe, what is the ratio of the weights of pineapple to grapes?

3. What fraction of the fruit salad is pear?

4. Jorge wants to bring 2 pounds total of this fruit salad to a potluck dinner. How much of each ingredient should he use?

5. Shawna only has 1 pound of grapes. How much of the other ingredients should she use?

Whole Group Reflection: 10 minutes
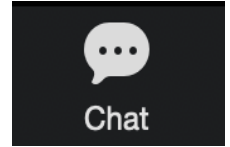
CHAT: Share any of your reflections on how MLR "Three Reads":

➢ Gave you access to language and mathematics?

■ We'll give you a minute to jot down some notes in your Participant Materials about Phase 6.

➢ What Mathematical Language Routines were modeled?
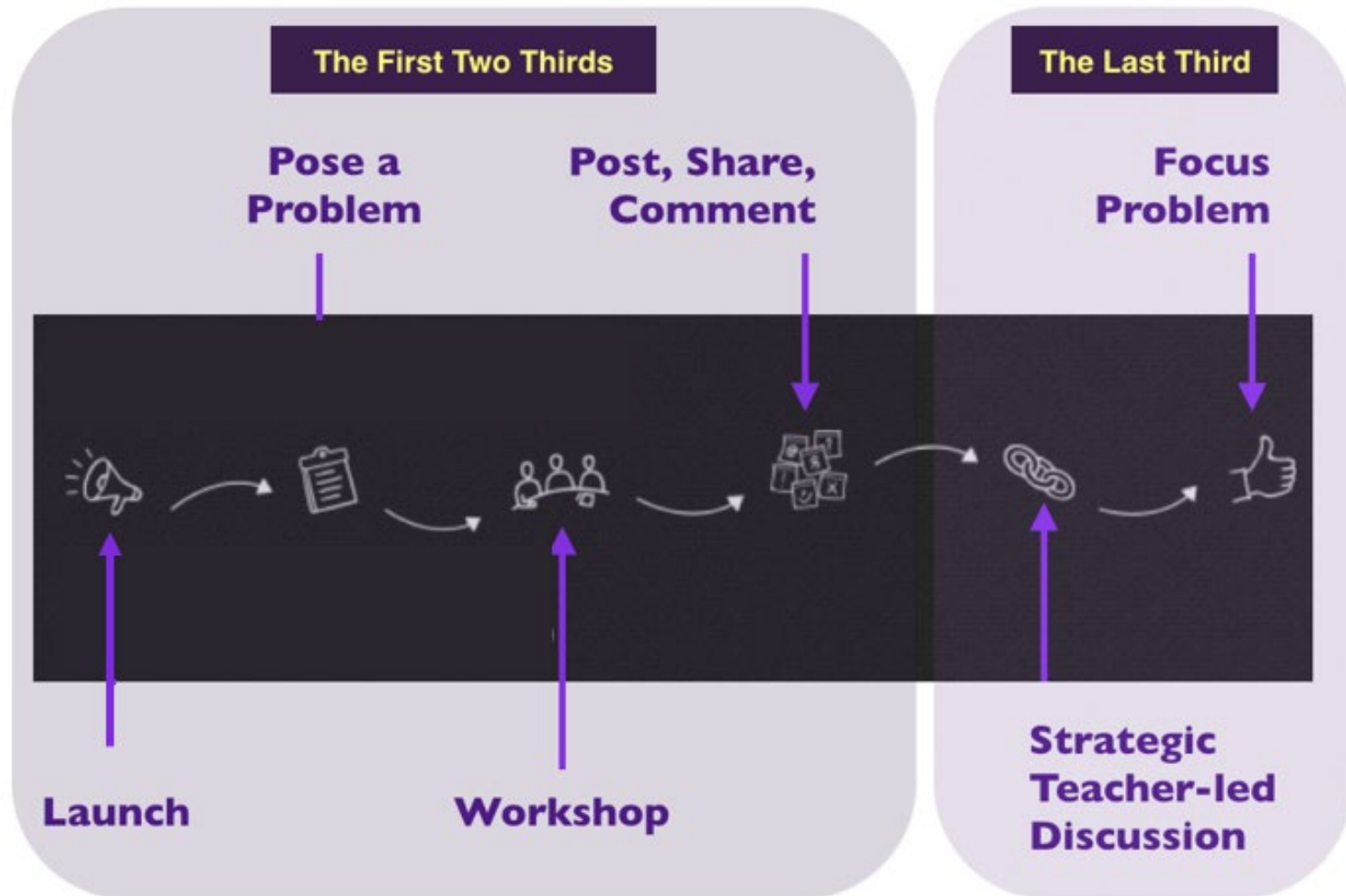
➢ What teacher moves and/or norms were used?

Reflecting on Connections Between the CCR and ELP Standards

- Write down some notes about how the connections between the Math Practices and the ELP Standards are evident in the model lesson.

- Share your connections in the CHAT.

DEBRIEFING MODEL LESSON AND ITS DESIGN (80 MINUTES)

Diagnostic Teaching: Six Phases

Diagnostic Teaching Lesson Design: Divergent and Convergent Thinking

| First Two Thirds of Lesson (Divergent Thinking)	Last Third of Lesson (Convergent Thinking)
The first two thirds is driven by a variety of learners' thinking: • Pose a compelling problem to learners that relates to critical mathematical concepts; • Give opportunities for learners to make their mathematical ways of thinking understandable to one another.	Last third focuses on pulling students' thinking together: • Use direct instruction near end of lesson when students are ready; • Summarize by quoting student work progressions and discussions from easiest to understand to most mathematically on level.

Reflect on the Diagnostic Teaching Design as a Whole: 5 minutes
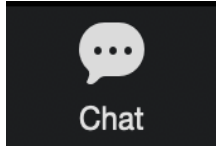
CHAT: Respond to the following prompts:

➤ How will the diagnostic teaching support English learners' agency, authority, and identity as a learner?

➤ How will the diagnostic teaching support English learners' academic language development?
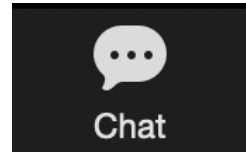
Team Time: 10 minutes

During the first two thirds of the lesson, we posed a problem and worked as a team (workshop). Then we posted, shared, and commented on one another's work.

1. Discuss this question in your group about the first two thirds of the model lesson:

   - How did the first two thirds of the lesson make the divergent ways of thinking public?

2. Be prepared to share answers to the questions in the chat when we reconvene as a whole group. (We will call on select teams to share verbally).

Whole Group Reflection on First Two Thirds of the Model Lesson: 15 minutes

CHAT: Share your answers to the question:

➢ How did the first two thirds of the lesson make the divergent ways of thinking public?

Team Time: 10 minutes

During the last third of the lesson, we facilitated the discussion toward pulling together—converging—students' thinking.

1. Discuss this question in your group about the last third of the lesson:

   - During the last third of the lesson, how did we make connections across representations, mathematics, and language?

2. Be prepared to share answers to the question in the chat when we reconvene as a whole group. (We will call on select teams to share verbally).

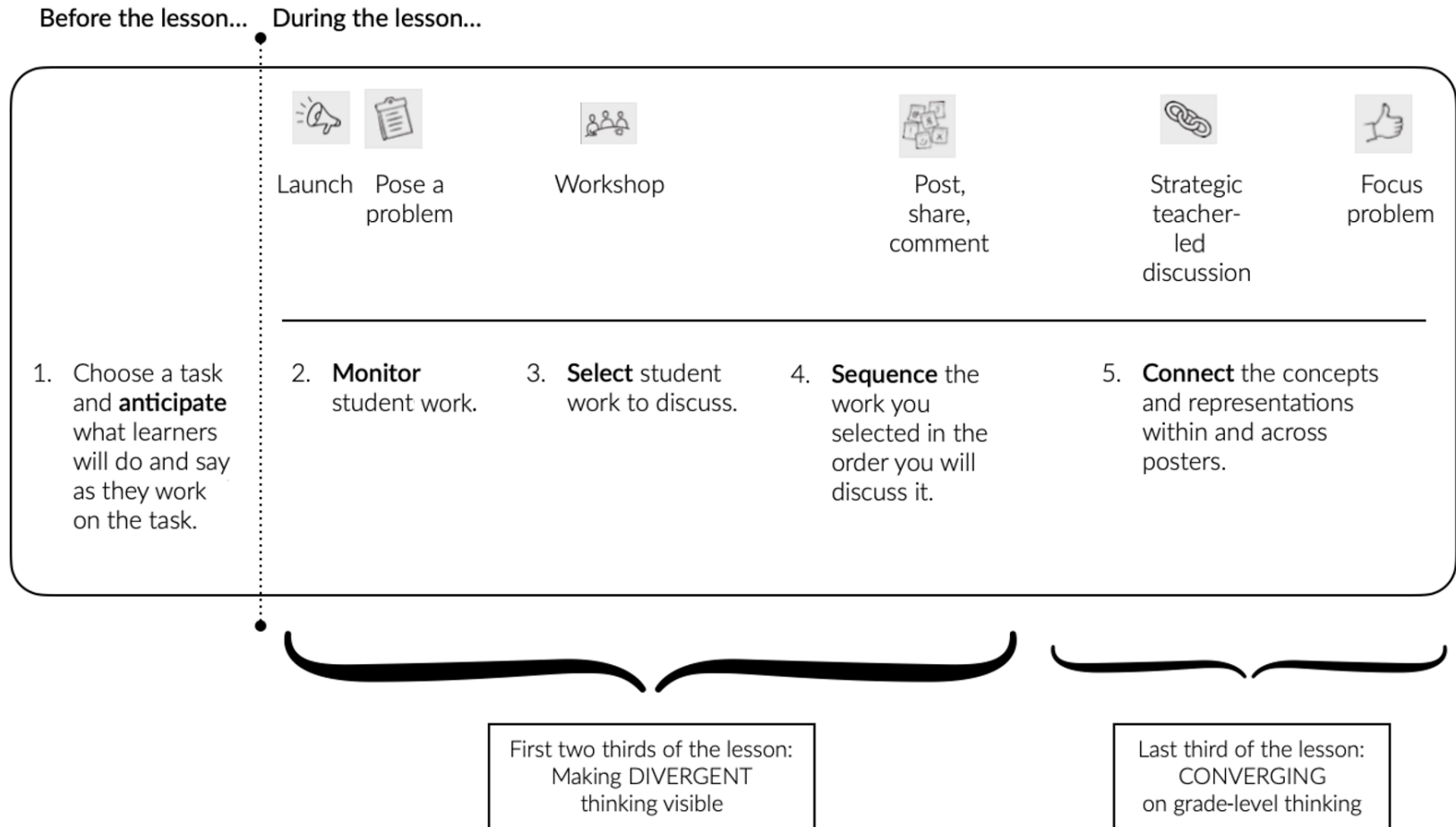# Whole Group Reflection on Last Third of the Model Lesson: 15 minutes
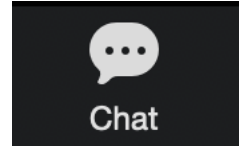
- CHAT: Share your answers to the question:

  ➢ During the last third of the lesson, how did we make connections across representations, mathematics, and language?
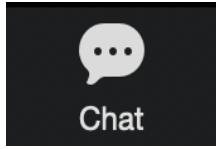
BREAK TIME (60 MINUTES)

Connecting to the Five Practices

The Five Practices for Orchestrating Productive Mathematics Discussions in a Lesson Design

Before the lesson...   During the lesson...

Launch   Pose a problem

Workshop

Post, share, comment

Strategic teacher-led discussion

Focus problem

1. Choose a task and **anticipate** what learners will do and say as they work on the task.

2. **Monitor** student work.

3. **Select** student work to discuss.

4. **Sequence** the work you selected in the order you will discuss it.

5. **Connect** the concepts and representations within and across posters.

First two thirds of the lesson: Making DIVERGENT thinking visible

Last third of the lesson: CONVERGING on grade-level thinking

Whole Group Reflection on Discussion Supports: 15 minutes

CHAT: Share any of your reflections on how MLR "Discussion Supports" helped orchestrate the discussion.

Putting It All Together: 10 minutes

- How is the concept of ratio used in your day-to-day life?

- What connections can we make to ELA? Add your comments to the Padlet.

- Jot down anything you don't want to forget.

Resource: Summary of Strategies, Routines, and Teacher Moves (Pages 19-25)

IMPLICATIONS FOR YOUR PRACTICE (30 MINUTES)

Team Time: 20 minutes

Question: Which language and mathematical routines do you think you can implement and how?

1. Individually, take a few minutes to write three to five key ideas you got today that you want to remember and use in your teaching. (5 minutes)

2. Decide on the three ideas that you wrote down of ways you might implement pedagogies or a routine. (15 minutes)

3. Each group should be prepared to share one example that was discussed.

Whole Group Debrief: 10 minutes
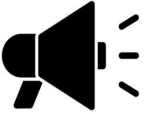
- Let's have each team share one way they might implement pedagogies or a routine.
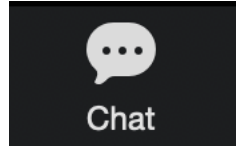
Thinking Back and Thinking Ahead

Take some time to think and jot down some answers for yourself:

- What connections can you make to what you learned in the previous training sessions?

- What do you feel confident you can use in your design for instruction?

- What will you try?

Wrap-Up

- Before we close, does anybody have any final questions or comments? Raise your hand or chat.

Thank you!