STATE-BASED CURRICULUM REVIEW FOR MATHEMATICS

PARTICIPANT WORKBOOK

2022

Standards-in-Action 2.0

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Introduction

The SIA 2.0 State-Based Curriculum Review Tool for Mathematics is organized by four research-based dimensions for mathematics:

1. Critical Mathematical Concepts and Skills
2. Mathematical Progressions and Connections
3. Reasoning and Communicating with Mathematics
4. Quality Mathematical Tasks

The dimensions provide a set of specific criteria for both mathematical content and for supports for English learners (ELs). Each content criterion includes a set of questions designed to guide your search for evidence in the curriculum. During this training, we will focus on the dimensions sequentially, starting with Dimension 1.

The Review Process

1. After reviewing the content criteria for a dimension, search the curriculum for evidence that each criterion is met. Following each content criterion, you will find a set of "guiding questions" that are included to help you better understand the intent of the criterion.
2. Place a checkmark next to each criterion for which you found evidence. In the appropriate section, cite the location(s) in the curriculum and provide a brief comment about the evidence that substantiates your checkmark (or lack thereof).
3. Then assign the content criteria a rating of 0, 1, or 2 points for each dimension.
4. Follow the same steps to rate the EL support criteria for each dimension.

Note some content criteria are indicated with an asterisk (*). They also are research-based EL supports. When rating for EL support, therefore, consider all the EL support criteria as well as the asterisked content criteria for the dimension.

Use this workbook to analyze the extent to which the provided model curriculum aligns to the evidence-based criteria of the curriculum review tool.

Use another copy of this workbook to assess the alignment of the curriculum from your state you have chosen to review. Your team lead will maintain a master copy of your team's consensus ratings and summary findings. Once your consensus review is complete, decide, in conjunction with your state leadership, how and with whom to share the review materials.

Dimension 1
Critical Mathematical Concepts and Skills

Review Content Criteria for Dimension 1:

1 ☐ **Content Criterion.** Curriculum supports the development of students' understanding of the most critical mathematical concepts required by the standards for the level.

- Are most of the lessons and assessments tied to critical mathematical concepts and skills required for the level?
- Does the curriculum support students in developing conceptual understanding of those critical concepts and skills?

Substantiation:

2 ☐ **Content Criterion.** Curriculum addresses supporting concepts in ways that enhance the focus on critical concepts.

- Do activities address supporting concepts to reinforce critical concepts?
- Are supporting concepts sufficiently minimized so as not to interfere with the overall focus on the most critical concepts?

Substantiation:

3 ☐ **Content Criterion.** Curriculum is designed so that students attain the required procedural skills and fluencies for the level.

- Does the curriculum provide sufficient practice opportunities for students to attain the computational and procedural fluencies for the level?
- Does the curriculum regularly assess students on the required fluencies?

Substantiation:

Dimension 1: Rating for Content Alignment

☐ 2 Most or all components of the content criteria are present.

☐ 1 Some components of the content criteria are present.

☐ 0 Few or no components of the content criteria are present.

Summary Comments:

Review EL Supports for Dimension 1.

1 ☐ **EL Support.** Curriculum explicitly points out the key mathematical concepts for each lesson.

- Do the student-facing materials state the concept target(s) for each lesson in clear and understandable terms?
- Are conceptual goals stated and explained for each lesson in the teacher-facing materials?

Substantiation:

2 ☐ **EL Support.** Curriculum identifies key mathematical terms that students need to know to understand concepts addressed for the level.

- Are the relevant terms, required for understanding the mathematics, identified for students?
- Are teachers encouraged to prepare students for the mathematics by identifying and defining key mathematical vocabulary words?

Substantiation:

3 ☐ **EL Support.** Curriculum includes a glossary or encourages the use of student-friendly dictionaries that define key vocabulary so students can look up unknown words.

- Are glossary definitions written in plain language, and do they include useful context or concrete examples?
- If there is no glossary, are appropriate resources suggested that can help students easily define unknown vocabulary?

Substantiation:

4 ☐ **EL Support.** Curriculum uses plain language that facilitates student access to the mathematical content required in the level.

- Are the examples and explanations in the student-facing materials written concisely and without jargon?
- Do teacher-facing materials model using clear, plain language that does not distract from the mathematics?

Substantiation:

Dimension 1: Rating for EL Supports

☐ 2 Most or all components of the EL supports are present.

☐ 1 Some components of the EL supports are present.

☐ 0 Few or no components of the EL supports are present.

Summary Comments:

Dimension 2
Mathematical Progressions and Connections

Review Content Criteria for Dimension 2:

1 ☐ **Content Criterion.** Curriculum explains ways for teachers and students to connect concepts level-to-level.

- Does the curriculum make explicit connections between the critical concepts of the level and those of future levels? For the teacher? For the student?
- Does the curriculum identify prerequisite knowledge that students will need in order to do the level-specific work?

Substantiation:

2 ☐ **Content Criterion.** Curriculum explains ways for teachers and students to connect concepts within the level. *

- Does the curriculum clarify the connections among the critical concepts within the level? For the teacher? For the student?
- Are teachers encouraged to introduce new concepts by connecting to what students already know?
- Does content build on understanding from previous lessons, as evidenced in the table of contents or sequence of lessons?

Substantiation:

3 ☐ **Content Criterion.** Curriculum devotes most attention to on-level content. Mathematics from previous levels is identified and does not interfere unduly with on-level content.

- Is mathematics content from previous levels clearly identified as “review”?
- Is the number of “review” lessons a small enough percentage of the whole so as not to distract from on-level content?
- Is the time spent reviewing past concepts purposeful and helpful, and does it support a deeper understanding of on-level content?

Substantiation:

Dimension 2: Rating for Content Alignment

☐ 2 Most or all components of the content criteria are present.

☐ 1 Some components of the content criteria are present.

☐ 0 Few or no components of the content criteria are present.

Summary Comments:

Review EL Supports for Dimension 2:

1 ☐ **EL Support**. Curriculum relates new mathematical vocabulary to knowledge students already have.

- When new mathematical terms are introduced, are they connected to concepts students previously learned?
- Are relationships clearly made between new vocabulary and words students may already know (e.g., “binomial” relates to “bicycle”)?

Substantiation:

Dimension 2: Rating for EL Supports *(include asterisked Content Criterion #2 in your rating)*

☐ 2 Most or all components of the EL supports are present.

☐ 1 Some components of the EL supports are present.

☐ 0 Few or no components of the EL supports are present.

Summary Comments:

Dimension 3
Reasoning and Communicating with Mathematics

Review Content Criteria for Dimension 3:

1 ☐ **Content Criterion.** Curriculum prompts students to produce mathematical arguments and to analyze the arguments of others. *

- Does the curriculum ask students to make mathematical claims and build a logical progression of statements to explore the truth of their claims?
- Are there requests in lessons, activities, and assessments for students to explain, show, or defend their findings?
- Are students asked to consider the merit of the mathematical reasoning of others?

Substantiation:

2 ☐ **Content Criterion.** Curriculum includes sufficient supports for teachers to provide opportunities for students to build their understanding of important mathematics through discussion and verbal engagement. *

- Are there suggestions in the teacher materials for strategic grouping so that students can build and share their mathematical thinking with others?
- Do the teacher materials suggest discussion questions that will elicit student interest and response?
- Are students encouraged to verbalize their thinking and given regular opportunities to practice speaking about the mathematics they are learning?

Substantiation:

3 ☐ **Content Criterion.** Curriculum prompts teachers and students to attend to the precision of their mathematical statements. *

- Does the curriculum encourage students to be precise when they use the content-specific language they are learning, both in speaking and in writing?
- Does the curriculum encourage teachers to repeat or rephrase students' statements to model precise mathematical language and to help students clarify their thinking?
- Does the curriculum include sufficient opportunities for students to see and experience examples of precise communication in mathematics?

Substantiation:

4 ☐ **Content Criterion.** Curriculum includes examples that demonstrate mathematical reasoning and a well-structured solution, without providing formulas for solving problems. *

- Do the examples from the curriculum demonstrate reasoning for a variety of problem types and problem-solving strategies?
- Do lessons lead students to a clearer understanding of the mathematics without providing step-by-step problem-solving recipes?

Substantiation:

5 ☐ **Content Criterion.** Curriculum promotes the strategic use of technology to support student reasoning.

- Does the curriculum encourage the use of technology as vital for mathematical curiosity and conceptual understanding?
- Does the curriculum provide opportunities and encouragement for students to use appropriate tools to assist in understanding and finding a solution to a problem?
- Are students taught when the calculator is most useful to employ and when mental calculations can be done more simply and efficiently?

Substantiation:

Dimension 3: Rating for Content Alignment

☐ 2 Most or all components of the content criteria are present.

☐ 1 Some components of the content criteria are present.

☐ 0 Few or no components of the content criteria are present.

Summary Comments:

Review EL Supports for Dimension 3:

1 ☐ **EL Support.** Curriculum regularly uses mathematical language routines to allow students to strengthen their language skills while engaging in on-level mathematics as defined by the standards.

- Does the curriculum encourage teachers to promote understanding through such instructional strategies as thoughtfully sequenced questions, strategic pairing of students, and multiple reads of text?
- Are students encouraged to ask questions and share their thinking, using both written and oral language, including use of home language or translanguaging? (Translanguaging is allowing use of a multilingual person's full language repertoire rather than keeping a narrow focus on a single language.)

Substantiation:

2 ☐ **EL Support.** Curriculum encourages teachers to use re-voicing to model correct mathematical language, to help students put their thoughts into words, and to clarify their responses.

- Are teachers encouraged to model accurate mathematical language for students?
- Do teacher-facing materials make specific suggestions for how students can be encouraged to precisely express their mathematical understanding verbally?

Substantiation:

3 ☐ **EL Support.** Curriculum provides acknowledgement of and ample support for students to learn specialized mathematical language, including attending to:

- Elements of mathematical words (e.g., prefixes, suffixes, and roots);
- High-value academic words (e.g., explain, interpret, etc.); and
- Multiple meanings of mathematical words (e.g., even, root, product, etc.).

Substantiation:

4 ☐ **EL Support.** Curriculum integrates language-based structures (e.g., linking phrases, sentence starters) to help students demonstrate their mathematical work and thinking.

- Does the curriculum offer techniques such as using sentence frames/starters, linking words, transitional phrases, and role-plays to help students express their mathematical understanding?
- Does the curriculum refer to learners' home languages as assets for learning academic content and English simultaneously?

Substantiation:

Dimension 3: Rating for EL Supports *(include asterisked Content Criteria #1–4 in your rating)*

☐ 2 Most or all components of the EL supports are present.

☐ 1 Some components of the EL supports are present.

☐ 0 Few or no components of the EL supports are present.

Summary Comments:

Dimension 4
Quality Mathematical Tasks

Review Content Criteria for Dimension 4:

1 ☐ **Content Criterion.** Curriculum includes a variety of high-quality tasks that are measurable and aligned to instructional objectives outlined in the standards. *

- Are there frequent opportunities for students to engage with sufficiently challenging tasks that apply significant and engaging mathematics?
- Do tasks involve all three aspects of rigor?
- Do tasks include opportunities for students to employ relevant Standards for Mathematical Practice?
- Do tasks provide sufficient guidance for students to complete them successfully and for teachers to administer and assess them effectively?

Substantiation:

2 ☐ **Content Criterion.** Curriculum includes application problems that are primarily on-level and that embody the critical concepts of the level.

- Are most application problems designed to promote and assess student understanding of the most critical mathematical concepts required for the level?
- Are application problems that address past or supporting concepts sufficiently minimized and clearly connected to the critical concepts for the level?

Substantiation:

3 ☐ **Content Criterion.** Curriculum suggests providing ample time for students to orient themselves to a problem and challenges them to make sense of problems without over scaffolding. *

- Do at least some problems, activities, and assessments require students to independently strategize and formulate problems and/or to defend their findings?
- Do the lessons provide examples that model reasoning for a variety of problem types, rather than providing problem-solving formulas?
- Do answer keys, rubrics, and scoring guidelines clearly connect to the requirements

of targeted standards and provide sufficient guidance for interpreting student performance?

Substantiation:

4 ☐ **Content Criterion.** Curriculum correctly portrays modeling as the application of mathematics to authentic problems that arise in everyday life, society, or the workplace. Students are expected to solve un-scaffolded modeling problems that require independent thinking and decision making.

- Are there application problems that require students to do more than simply use manipulatives, explain their process, or convert between representations?
- Are there opportunities for students to solve authentic, real-world problems that require at least some modeling with mathematics?
- Are there opportunities for students to work and think independently?

Substantiation:

5 ☐ **Content Criterion.** Curriculum encourages students to produce multi-modal representations of terms and concepts when solving problems and justifying solutions. *

- Do the demands of instructional tasks and applications regularly ask students to show their thinking in multiple ways (e.g., in words and a drawing or graph)?
- Do lessons regularly include examples that demonstrate multiple ways to represent mathematical concepts, both during the problem-solving process and in presenting solutions?

Substantiation:

Dimension 4: Rating for Content Alignment

☐ 2 Most or all components of the content criteria are present.

☐ 1 Some components of the content criteria are present.

☐ 0 Few or no components of the content criteria are present.

Summary Comments:

Review EL Supports for Dimension 4:

1 ☐ **EL Support.** Curriculum encourages the teacher to present oral and visual directions, to repeat the directions, and to check for understanding by asking students to restate them.

- Do the teacher-facing materials include suggestions for presenting directions for a task in multiple ways?
- Are teachers encouraged to check for student understanding of directions for a task or activity?

Substantiation:

2 ☐ **EL Support.** Curriculum offers teachers guidance on how to provide explicit, constructive, and targeted feedback on student work.

- Does the curriculum include suggestions for assessing student understanding in a variety of ways?
- Are teachers encouraged to use constructive feedback for students to move them toward a better understanding of the mathematics?

Substantiation:

Dimension 4: Rating for EL Supports *(include asterisked Content Criteria #1, #3, and #5 in your rating)*

☐ 2 Most or all components of the EL supports are present.

☐ 1 Some components of the EL supports are present.

☐ 0 Few or no components of the EL supports are present.

Summary Comments:

Overall Ratings and Recommendations

Determining an Overall Rating for Content Alignment: Add the total points assigned to each dimension's content criteria. The highest possible score would be 8 points (or 2 points per dimension).

Determining an Overall Rating for EL Supports: Add the total points assigned to each dimension's EL support criteria. The highest possible score would be 8 points (or 2 points per dimension).

Summarizing Comments and Recommendations: Summarize the key strengths and weaknesses of the curriculum you reviewed. Then document any recommendations regarding the continued use of the curriculum in your program(s).

Remember, it is unlikely that any curriculum will contain every content criterion and every EL support included in this review tool. Look specifically at what is missing and consider:

- How important are the missing criteria to your overall rating?
- Is their absence so critical that you cannot assign an overall rating of Well Aligned or Well Supported?
- Alternatively, how easily could you and your fellow adult educators fill in the gaps in the curriculum?

Curriculum Reviewed:________________________ Content Standards Level: ______

Reviewer(s):________________________________ Date(s) of Review: __________

Description of the Curriculum Sample Reviewed:

Overall Ratings, Summary Comments, and Recommendations

Overall Rating: Content Alignment	Overall Rating: English Learner Supports
☐ Well Aligned (6–8 points)	☐ Well Supported (6–8 points)
☐ Somewhat Aligned (3–5 points)	☐ Somewhat Supported (3–5 points)
☐ Not Aligned (0–2 points)	☐ Not Well Supported (0–2 points)

Summary Comments and Recommendations:

Appendices A–D

Appendix A: Critical Concepts and Fluencies of the Level

COLOR KEY:

Black – Number
Red – Algebra
Blue – Geometry
Green – Statistics and Probability

LEVEL A
Critical Concepts of the Level Developing understanding of whole number place value for tens and ones Developing understanding of addition and subtraction, and the properties of these operations Describing and reasoning about shapes and their attributes Developing understanding of linear measurement
Fluencies of the Level Adding and subtracting within 10
Supporting Concepts of the Level Analyzing, comparing, creating and composing geometric shapes Classifying and counting objects in different categories

LEVEL B
Critical Concepts of the Level Extending understanding of base-10 notations Adding and subtracting to 1,000; fluency and application to 100 Understanding multiplication and division of whole numbers Understanding division as the inverse of multiplication; single-digit divisors Developing understanding of fractions, especially unit fractions Using standard units of measure for length, time, liquid volume and mass Developing understanding of area and its relationship to addition and multiplication Analyzing and partitioning two-dimensional shapes
Fluencies of the Level Adding and subtracting within 1,000 (from memory within 100) Multiplying and dividing within 100
Supporting Concepts of the Level Working with equal groups of objects (foundations for multiplication) Working with time and money Reasoning about shapes and their attributes Representing and interpreting data

LEVEL C

Critical Concepts of the Level

Extending the number system to positive rational numbers
Extending place value understanding for decimals to thousandths
Attaining fluency with operations, using multi-digit whole numbers and decimals
Understanding fraction equivalence and comparison
Developing fluency with sums and differences of fractions
Connecting ratio and rate to whole number multiplication and division
Writing, evaluating and interpreting expressions and equations
Developing understanding of the coordinate planes
Classifying geometric two-dimensional figures based on properties
Developing understanding of and solving problems involving volume and surface area
Developing understanding of statistical variability

Fluencies of the Level

Adding/subtracting whole numbers within 1,000,000
Performing multi-digit multiplication (from memory within 100)

Supporting Concepts of the Level

Working with factors and multiples
Converting among units within a measurement system
Representing and interpreting data

LEVEL D

Critical Concepts of the Level

Extending number sense and fluency with operations to all rational numbers
Understanding ratio and rate and using them to solve problems
Applying proportional relationships
Working with expressions and linear equations
Solving linear equations and systems of linear equations
Developing the concept of functions
Graphing functions in the coordinate plane and analyzing the graphs
Solving problems involving scale drawings
Solving problems involving two- and three-dimensional figures: area, surface area and volume
Analyzing two- and three-dimensional shapes using side length and angle measurements, similarity and congruence
Applying the Pythagorean theorem
Understanding patterns of association for bivariate data and describing them with a linear equation, when appropriate
Summarizing and interpreting data and data distributions
Understanding and applying probability concepts
Drawing inferences about populations based on random samples (probability distributions)

Fluencies of the Level

Performing multi-digit division and decimal operations
Solving linear equations of the form, px + q = r and p(x + q) = r
Solving simple 2x2 systems by inspection

Supporting Concepts of the Level

Using random sampling to draw inferences about a data population
Investigating chance processes and developing and using probability models
Investigating patterns of association in bivariate data

LEVEL E

Critical Concepts of the Level

Extending understanding of number systems to the set of real numbers
Writing equivalent expressions involving radicals and rational exponents
Reasoning quantitatively; using appropriate units and levels of precision
Defining, evaluating, comparing and modeling with linear, quadratic and exponential functions and equations
Building, interpreting and analyzing functions using different representations
Reasoning with and solving linear, quadratic and exponential equations and linear inequalities
Interpreting and using the structure of expressions to solve problems
Operating with algebraic expressions, including polynomials and rational expressions
Applying similarity and congruence concepts to geometric figures, including triangles
Using geometric models and volume formulas to solve measurement problems
Summarizing, representing and interpreting one- and two-variable data, including using frequency tables

Fluencies of the Level

Performing rational number operations
Adding, subtracting and multiplying with polynomials
Transforming expressions, using algebraic calculations (grouping, factoring, etc.)
Solving linear equations

Supporting Concepts of the Level

Understanding and applying inverse functions, including logarithmic functions
Defining trigonometric ratios on the unit circle
Modeling with trigonometric functions and their graphs
Understanding and applying conic sections

Appendix B: Critical Mathematical Concepts That Progress Across the Levels

Number and Operations:

- Level A – Base 10 addition and subtraction (2-digit); multiples of 10
- Level B – Base 10 addition and subtraction (3-digit) and multiplication (1-digit by multiples of 10); Understanding and representing fractions
- Level C – Base 10 and decimal addition, subtraction, multiplication and division (multi-digit); Operating and comparing fractions
- Level D – Rational number addition, subtraction, multiplication and division (multi-digit); Ratio and rate reasoning
- Level E – Real number addition, subtraction, multiplication and division (multi-digit)

Algebraic Thinking (Expressions, Equations and Functions):

- Level A – Using a variable to represent an unknown value
- Level B – Representing and solving problems using expressions and equations
- Level C – Writing, reading and evaluating expressions; representing and solving equations; writing an inequality
- Level D – Graphing and solving linear equations; solving pairs of simultaneous equations; understanding functions
- Level E – Interpreting expressions, equations and inequalities; understanding polynomials; solving linear, quadratic, simple rational and radical equations; using and interpreting linear, quadratic and exponential functions; graphing functions

Geometry and Measurement:

- Level A – Analyzing and composing two-dimensional shapes; understanding length measurements
- Level B – Categorizing shapes; partitioning; measuring and estimating length; measuring time and liquid volume; finding rectangular area
- Level C – Understanding and using the coordinate plane; drawing polygons; understanding angles and angle measures; measuring perimeter and area (quadrilaterals and right triangles) and volume (right rectangular prisms); converting among measurement systems
- Level D – Measuring the circumference and area of circles; understanding congruence and similarity; understanding the relationships of sides and angles in a triangle; applying the Pythagorean theorem
- Level E – Defining parallel and perpendicular; finding volumes of cylinders, cones, pyramids and spheres

Data, Statistics and Probability:

- Level A – Organizing and representing data
- Level B – Generating and graphing data
- Level C – Finding and interpreting measures of center; representing data with dot plots, box plots and histograms
- Level D – Summarizing data in a context; interpreting data using measures of center and variability; understanding and developing probability models; solving probability problems; constructing and interpreting scatterplots; displaying frequencies and relative frequencies in a two-way table
- Level E – Interpreting differences in center and spread in the context of a data set; summarizing data in a two-way frequency table; interpreting rates of change in data; distinguishing between correlation and causation

Appendix C: Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs; draw diagrams of important features and relationships; graph data and search for regularity or trends. Less experienced students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand different approaches to solving complex problems and identify correspondences between the approaches.

MP.2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: (1) the ability to decontextualize — to abstract a given situation, represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents; and (2) the ability to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, rather than just computing with them; and flexibly using different properties of operations and objects.

MP.3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They can analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them and respond to the arguments of others. They reason inductively about data, making plausible arguments that consider the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain it. Less experienced students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not initially generalized or made formal. Later, students learn to determine domains to which a mathematical assumption applies. Students at all levels can listen to or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MP.4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. This might be as simple as writing an addition expression or equation to describe a situation. A student might apply proportional reasoning to plan a school event or analyze a problem in the community. A student might use geometry to

solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically and draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are familiar with the appropriate tools for their course and are able to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various levels can identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and are able to recognize situations where working without technology is more efficient.

MP.6. Attend to precision.
Mathematically proficient students can communicate precisely to others. They use precise mathematical vocabulary in discussion with others and in their own reasoning. They understand the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the context of the problem. Less experienced students are able to articulate carefully formulated explanations to each other. By the time they reach the higher levels, they have learned to examine claims and make explicit use of mathematical definitions.

MP.7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Students in early levels, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see that 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and a shift of perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP.8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look for both general methods and shortcuts. Early on, students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel out when expanding $(x–1)(x+1)$, $(x–1)(x^2+x+1)$ and $(x–1)(x^3 +x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix D: Determine if a Well-Aligned Curriculum Is Also a Good Fit for Your Program

Consider the following indicators only if your review has resulted in a rating of Well Aligned and the curriculum will support your English learners (ELs). These additional indicators will help you determine if this curriculum is a *good fit* for your program. The more that apply, the better the fit for you.

1. ☐ Has evidence of positive student outcomes.
2. ☐ Is affordable.
3. ☐ Includes educative features and support materials for instructors.
4. ☐ Reflects research and best practices.
5. ☐ Is appropriate for adult education purposes.
6. ☐ Appears to be relatively easy for all instructors to learn and implement.
7. ☐ Includes easily adaptable lessons for all types of adult learners, including student populations with specific needs.