CULTIVATING A LANGUAGE AND CONTENT FOCUS FOR ENGLISH LEARNERS

Mathematics Participant Materials: Ratios and Proportions

Standards-in-Action

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Section 1: Notes

NOTES ON MODEL LESSON

As you are engaging in the lesson, note any language supports or pedagogical structures the facilitator engages in. Just take note of what you experience that feels or seems to be a language support or pedagogical structure as defined below.

***Language supports**—scaffolds, structures, or routines that improve the precision and logic of the language used to communicate and reason in academic conversations.*

***Pedagogical structures**—teacher moves, practices, or routines that deepen student's understanding of the content they are learning.*

PHASE 1: LAUNCH THE LESSON

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

PHASE 2: POSE A PROBLEM

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

PHASE 3: WORKSHOP

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

PHASE 4: POST, SHARE, COMMENT

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

PHASE 5: STRATEGIC TEACHER-LED DISCUSSION

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

PHASE 6: FOCUS PROBLEM

LANGUAGE SUPPORTS (Mathematical Language Routines):

PEDAGOGICAL STRUCTURES (Socio-Mathematical Norms, Teacher Moves):

GENERAL NOTES:

Section 2: Directions for Team Time

NORMS TO PROMOTE LEARNING

TEAM TIME: 10 MINUTES

DIRECTIONS

1. Discuss how you think these socio-mathematical norms promote learning:
    - Errors are gifts; they promote discussion and learning.
    - The answer is as important as the mathematical thinking.
    - Ask questions until ideas make sense.
    - Think with language and use language to think.
    - Use multi-modal communication.
2. Be prepared to share comments or questions in the chat when we reconvene as a whole group. (We will call on select teams to share verbally).

MATHEMATICAL LANGUAGE ROUTINES JIGSAW

TEAM TIME: 20 MINUTES

DIRECTIONS

1. Divide your state team into four subgroups. Each subgroup will read two MLRs (pages 23-25). Note what the MLR is and why it is useful and be ready to share. (5 minutes)

    - Subgroup 1 – MLR 1 and MLR 2
    - Subgroup 2 – MLR 3 and MLR 4
    - Subgroup 3 – MLR 5 and MLR 6
    - Subgroup 4 – MLR 7 and MLR 8

2. Share within your state groups. (10 minutes)

PHASE 1: LAUNCH THE LESSON

TEAM TIME: 25 MINUTES

DIRECTIONS

1. Individually, write down possible mathematical questions that might be asked about the situation, any missing information, or assumptions that are important. (2 minutes)

2. Share your questions and review your team's questions, with some brief discussion in team time. (8 minutes)

3. Agree on what questions to copy and select one person to add the questions to the Padlet. (5 minutes)

4. Debrief: Look across at the questions from the other teams in the Padlet. (10 minutes)

PHASE 2: POSE A PROBLEM

TEAM TIME: 20 MINUTES

DIRECTIONS

1. Individually, fill in the missing values on Worksheet 1 (on page 16 of the Participant Materials).

2. In Team Time (15 minutes):

- Agree on the values you have created.
- Discuss three questions:
  - Which parts of this table always stayed the same?
  - Why do they stay the same?
  - Which parts changed and why?

2. Be prepared to share the answers to Worksheet 1 and the questions when we reconvene as a whole group.

PHASE 3: WORKSHOP

TEAM TIME: 40 MINUTES

DIRECTIONS

1. Your team will create a new recipe for chocolate milk different from Steven's. You choose the intensity.

2. You will collaborate on a Jamboard to describe how much chocolate syrup and how much milk to use for three different situations.

3. For Situation A and Situation B on Worksheet 2 (page 18), use the same recipe. Modify your recipe for Situation C for someone who likes less-intense chocolate milk.

4. Make a poster of your new recipe to share with the whole group when we reconvene.

- After you choose your recipe, make a poster that your classmates can understand.
- Your poster should include multiple representations (e.g., drawings, diagrams, tables, equations, graphs) to show how you used your recipe in each situation.
- Include both the ratio of chocolate syrup and milk, and the number of ounces of each ingredient for your recipe.
- Show all your steps. Which parts changed?

PHASE 6: FOCUS PROBLEM

TEAM TIME: 20 MINUTES

DIRECTIONS

1. Individually, work on your answers to Worksheet 3 (page 18). (5 minutes)

2. Discuss the answers and make sure the group agrees on the answers. (15 minutes)

3. Select a volunteer to share your answers to the questions—if called upon—when we reconvene as a whole group. (We will call on a selection of teams to share.)

DEBRIEFING THE MODEL LESSON AND ITS DESIGN: PART 1

TEAM TIME: 10 MINUTES

DIRECTIONS

1. Discuss this question in your group about the first two thirds of the model lesson:

    - How did the first two thirds of the lesson make the divergent ways of thinking public?

2. Be prepared to share answers to the questions in the chat when we reconvene as a whole group. (We will call on select teams to share verbally.)

DEBRIEFING THE MODEL LESSON AND ITS DESIGN: PART 2

TEAM TIME: 10 MINUTES

DIRECTIONS

1. Discuss this question in your group about the last third of the lesson:

    - During the last third of the lesson, how did we make connections across representations, mathematics, and language?

2. Be prepared to share answers to this question in the chat when we reconvene as a whole group. (We will call on select teams to share verbally.)

IMPLICATIONS FOR YOUR PRACTICE

TEAM TIME: 20 MINUTES

DIRECTIONS

Question: Which language and mathematical routines do you think you can implement and how?

1. Individually, take a few minutes to write three to five key ideas you got today that you want to remember and use in your teaching. (5 minutes)

2. Decide on the three ideas that you wrote down of ways you might implement pedagogies or a routine. (15 minutes)

3. Each group should be prepared to share one example that was discussed.

Section 3: Worksheets

Steven's Award Winning Chocolate Milk Recipe

Instructions

Steven made chocolate milk three times this week. He always followed his recipe, but changed the amount of chocolate milk. The first time he used 4 ounces of syrup. The second time he used 1 quart of milk. The third time (for the party) he made a total of 6 gallons of chocolate milk. Complete the table for the missing parts.

Mix:
1 part chocolate syrup 2 parts cold milk

Steven's Recipe

	Chocolate Syrup	Milk	Chocolate Milk	Ratio Syrup: Milk	Ratio Syrup: Chocolate Milk	Ratio Milk: Chocolate Milk
1	4 oz					
2		1 quart				
3			6 gallons			

Student Name: _____

The Intensity of Chocolate Milk

Create your own Recipe – Worksheet

2

Instructions

Create your own recipe for chocolate milk with your group-mates. The intensity of your recipe should be different from Steven's recipe.

Chocolate milk for just yourself.
Use your recipe to make one 8-ounce glass of chocolate milk.

Situation A

Chocolate milk for a group.
Use your recipe to make four 6-ounce glasses of chocolate milk.

Situation B

Chocolate milk for your grandma.
She likes very weak (not intense) chocolate milk. Change your recipe to make an 8-ounce glass of chocolate milk for your grandma.

Situation C

Recipe A	Recipe B	Recipe C

Student Name: _____

The Intensity of Chocolate Milk

From Chocolate Milk to Fruit Salad – Worksheet 3

The Seesaw County Fair: Award-Winning Fruit Salad

Instructions

Felicia made the winning fruit salad at the Seesaw County Fair. In her recipe, she used 4 pounds of pears, 3 pounds of grapes, 6 pounds of pineapples, and 3 pounds of oranges. Answer the following questions using Felicia's winning fruit salad recipe.

1. Based on this recipe, what is the ratio of the weights of oranges to pears?

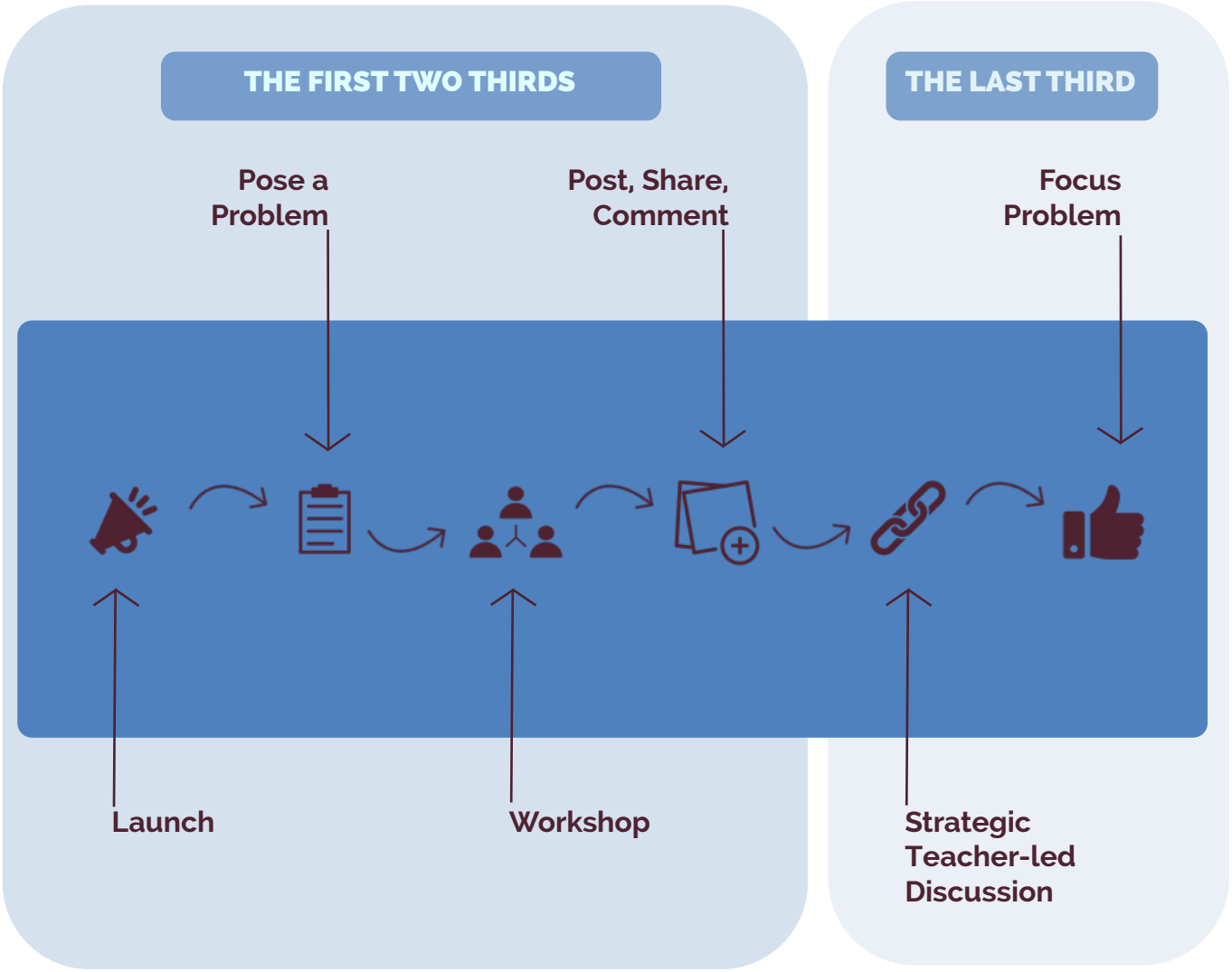
2. Based on this recipe, what is the ratio of the weights of pineapples to grapes?

3. What fraction of the fruit salad is pear?

4. Jorge wants to bring 2 pounds total of this fruit salad to a potluck dinner. How much of each ingredient should he use?

5. Shawna only has 1 pound of grapes. How much of the other ingredients should she use?

Section 4: Summary of Strategies, Routines, and Teacher Moves



MATHEMATICAL LANGUAGE ROUTINES

A 'math language routine' refers to a structured but adaptable format for amplifying, assessing, and developing students' language.

MLR1: Stronger and Clearer Each Time

MLR2: Collect and Display

MLR3: Critique, Correct, and Clarify

MLR4: Information Gap

MLR5: Co-Craft Questions and Problems

MLR6: Three Reads

MLR7: Compare and Connect

MLR8: Discussion Supports

SOCIO-MATHEMATICAL NORMS

- **Errors are gifts...they promote discussion and learning**

- **The answer is important, but not only the math!**

- **Ask questions...until ideas make sense.**

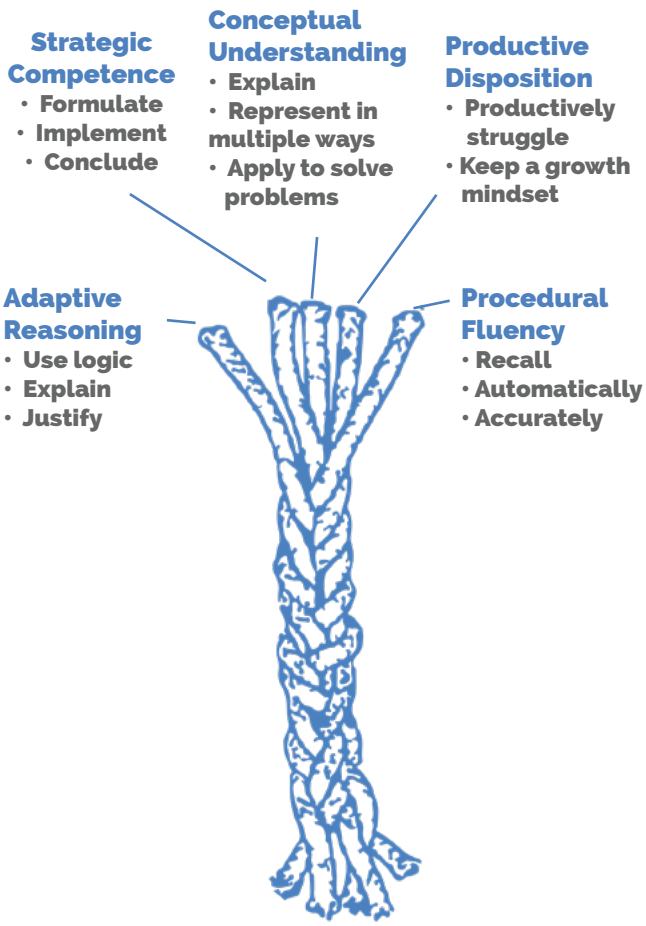
- **Think with language... use language to think.**

- **Use multi-modal communication.**

FIVE TEACHER MOVES

- **Just in time, not just in case.** Give students two or three instructions and get them started. Fix what breaks.
- **Students are the audience for students.** Students draft and revise explanations for other students to understand (not for the teacher).
- **"Everyone ready."** Students prepare to explain their thinking to other students (not just volunteers who go "oooh...ooh...ooh").
- Use **"make an expert"** then **"turn and talk"** when productive struggle weakens, and to focus on target mathematics.
- **Use direct instruction near END of lesson,** when students are ready. Summarize quoting student work progression.

INTERTWINED STRANDS OF PROFICIENCY



FIVE PRACTICES FOR ORCHESTRATING PRODUCTIVE MATHEMATICS DISCUSSIONS

by Margaret Smith & Mary Key Stein

0. Setting Goals and Selecting Tasks

1. Anticipating
(e.g., Fernandex & Yoshida, 2004; Schoenfeld, 1998)

2. Monitoring
(e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)

3. Selecting
(e.g., Lampert, 2001; Stigler & Hiebert, 1999)

4. Sequencing
(e.g., Schoenfeld, 1998)

5. Connecting
(e.g., Ball, 2001; Brendehur & Frykholm, 2000)

Left: National Research Council, & Mathematics Learning Study Committee.(2001). Adding it up: Helping children learn mathematics. National Academies Press.

Right: Smith, M.S., & Stein, M.K. (2011). Five practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.

THREE P'S OF T'N'T

- **Prompt**
- **Purpose**
- **Product**

QUESTIONS

- **Assessing**
- **Orienting**
- **Advancing**

THREE READS

What is the situation about?

What are all the quantities in this situation?

What are all the mathematical questions we could ask of this situation?

TALK MOVES

- **Turn and Talk**
- **Say More**
- **Revoicing**
- **Who can repeat?**
- **Press for Reasoning**
- **What do you think about that?**
- **Do you agree or disagree? Why?**

A ROUTINE FOR TALKING

- **Solo**
- **Partner**
- **Small Group**
- **Whole Class**

Left, upper:
Driscoll, M. (1999) Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912.

Left, lower:
http://bit.ly/2InPbTS_3READS

Right, upper:
Chapin, S.H., O'Conner, C., O'Conner, M.C., & Anderson, N.C. (2009). Classroom discussions: Using math talk to help students learn, Grades K-6. Math Solutions.

EIGHT MATHEMATICS LANGUAGE ROUTINES

STRONGER AND CLEARER EACH TIME (MLR1)

What: Students write a first draft response to a prompt, then engage in successive pair-shares to have multiple opportunities to refine and clarify their response through conversation, and then finally revise their original response. Throughout this process, students should be encouraged to press each other for clarity and details.

Why: Stronger and Clearer Each Time (MLR1) provides a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. The routine provides a purpose for student conversation and fortifies student output.

COLLECT AND DISPLAY (MLR2)

What: The teacher captures students' oral words and phrases into a stable, collective reference in order to stabilize the fleeting language that students use during partner, small-group, or whole-class activities. The teacher listens for, and scribes, the student output using written words, diagrams and pictures; this collected output can be organized, revoiced, or explicitly connected to other language in a display for all students to use over the course of a lesson or unit.

Why: Collect and Display (MLR2) provides feedback for students in a way that increases accessibility while simultaneously supporting meta-awareness of language. The routine mirrors student language back to the whole class to enable students' own output to be used as a reference in developing their mathematical language over time.

CRITIQUE, CORRECT, CLARIFY (MLR3)

What: Students are given a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The writing is an incorrect, incomplete, or unclear 'first draft' written argument or explanation, and students' job is to improve the writing by correcting any errors, clarifying meaning, and adding explanation, justification, or details. The routine begins with a brief critique of the first draft in which the teacher elicits 2-3 ideas from students to identify what could use improvement, then students individually write second drafts, and finally the teacher scribes as 2-3 students read their second drafts aloud.

Why: Critique, Correct, Clarify (MLR3) prompts student reflection,fortifies output and builds students' meta-linguistic awareness. The final step of public scribing creates an opportunity to invite all students to help edit a final draft together, so that it makes sense to more people. Teachers can demonstrate with meta-think-alouds and press for details when necessary.

INFORMATION GAP (MLR4)

What: This routine allows teachers to facilitate meaningful interactions by positioning some students as holders of information that is needed by other students. Teachers give partners or team members different pieces of necessary information that must be used together to solve a problem or play a game.

Why: With an information gap (MLR4), students need to orally (and/or visually) share their ideas and information in order to bridge the gap and accomplish something that they could not have done alone. Teachers should model how to ask for and share information, clarification, justification, and elaboration. This routine cultivates conversation.

CO-CRAFT QUESTIONS (MLR5)

What: Students are presented with a picture, video, diagram, data display, or description of a situation, and their job is to generate one or more mathematical questions that could be asked about the situation. Students then share and compare their questions, as the teacher calls attention to questions that align with the content goals of the lesson. Finally, the "official" question or problem is revealed for students to work on.

Why: Co-Craft Questions (MLR5) allows students to get inside of a context before feeling pressure to produce answers, and creates space for students to produce the language of mathematical questions themselves. Use this routine to spark curiosity about a new mathematical idea or representation, and to elicit everyday student language to brainstorm about the quantitative relationships that might be investigated. During this routine, students use conversation skills and develop meta-awareness of the language used in mathematical questions and problems.

THREE READS (MLR6)

What: A word problem is read three times, with a different question posed with each read: (1) What is this situation about? (2) What can be counted or measured in this situation? (3) What mathematical questions could we ask?

Why: MLR6 Three Reads gives students a chance to use everyday language to help each other make sense of the context -- and the language -- of a word problem before jumping down a solution path. Use this routine to ensure that students know what they are being asked to do, and to create an opportunity for students to reflect on the general structure of quantitative situations and on the ways mathematical questions are presented. This routine supports reading comprehension of problems and meta-awareness of mathematical language. It also supports negotiating information in a text with peers through mathematical conversation.

COMPARE AND CONNECT (MLR7)

What: The teacher facilitates a discussion about two or more pieces of student work that include distinct mathematical representations or approaches to a problem, calling attention to the correspondences among quantities, relationships, and features of the representations. Teachers should demonstrate thinking out loud (e.g., exploring why we one might do or say it this way, questioning an idea, wondering how an idea compares or connects to other ideas or language), and students should be prompted to reflect and respond.

Why: Use Compare and Connect (MLR7) to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, and language. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

DISCUSSION SUPPORTS (MLR8)

What: The teacher uses multi-modal strategies for helping students comprehend and generate language and ideas, such as sentence frames, word walls, images and videos, revoicing, choral response, gesture, and graphic organizers. The strategies can be combined and used together with any of the other routines.

Why: Discussion Supports (MLR8) foster rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies. Use Discussion Supports to make classroom communication accessible, to increase meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas. [1]

1 Adapted in 2021 by Vinci Daro in collaboration with Barbara Beske for Charlotte-Mecklenburg Schools, based on previous work with Understanding Language/Stanford Center for Assessment, Learning and Equity, and Illustrative Mathematics